Variable Clark Unit Hydrograph Parameter Regression Equations for California

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Abstract

The Clark unit hydrograph transform (Clark, 1945) is the most commonly employed runoff transform method within hydrologic modeling applications undertaken by the U.S. Army Corps of Engineers (USACE). This method is mature, well established, well documented, and simple to set up and use. Also, parameters can be regionalized, related to measurable basin characteristics, and varied with excess-precipitation rates, which are important traits for methods used in dam safety studies.

The new Variable Clark unit hydrograph method contained within the Hydrologic Engineering Center's (HEC) Hydrologic Modeling System (HEC-HMS) builds upon the "classical" Clark unit hydrograph method by allowing parameters (e.g., time of concentration and storage coefficient) to change throughout a simulation. This new method allows modelers to efficiently simulate the nonlinear dynamic runoff response of a watershed when subjected to large excess precipitation rates, such as those expected during design storms like the Probable Maximum Precipitation (PMP). The California Department of Water Resources, Division of Safety of Dams (DSOD) partnered with HEC to develop methods to estimate Variable Clark unit hydrograph parameters throughout the state of California for use within dam safety applications. Through the use of these equations, synthetic unit hydrograph parameters can be quickly derived for independent watersheds when observed data is not readily available.

New tools within HEC-HMS were used to construct numerous hydrologic models, develop initial parameter estimates, and calibrate parameters using observed data. Regression equations that relate physically measurable watershed characteristics (e.g., longest flow path) to Variable Clark unit hydrograph parameters (e.g., time of concentration and storage coefficient vs. excess precipitation rates) were then developed. Predicted Variable Clark unit hydrograph parameters obtained using the aforementioned regression equations were then evaluated using a validation process. The results of this validation process demonstrated that each regression equation adequately predicted Variable Clark unit hydrograph parameters throughout the state of California.

Introduction

Clark Unit Hydrograph Transform

The Clark unit hydrograph method utilizes the concept of an instantaneous unit hydrograph to route excess precipitation to the subbasin outlet. An instantaneous unit hydrograph is derived by instantaneously applying a unit depth (e.g., one inch) of excess precipitation over a watershed (Clark, 1945). This method explicitly represents two critical processes in the transformation of excess precipitation to runoff: 1) the translation (or movement) of excess precipitation from its origin throughout the watershed to the outlet and 2) the attenuation (or reduction) of the magnitude of the discharge as the excess precipitation is temporarily stored throughout the watershed (U.S. Army Corps of Engineers, 2021). These two processes are explicitly incorporated to estimate the hydrograph at the watershed outlet. Runoff is first translated to the watershed outlet with delay but without attenuation. Attenuation is then applied at the watershed outlet. Three parameters are utilized within this method:

- Time of concentration (T_c), which is equivalent to the time it takes for excess precipitation to travel from the hydraulically-most remote point of the watershed to the outlet,
- Watershed storage coefficient (R), which is equivalent to attenuation due to storage effects throughout the watershed (Kull & Feldman, 1998), and
- Time-Area histogram, which represents the watershed area that contributes to flow at the outlet as a function of time.

This method is mature, well established, well documented, and simple to set up and use. Also, parameters can be regionalized, related to measurable basin characteristics, and varied with excess-precipitation rates, which are important traits for methods used in dam safety studies. A previous study developed regional regression equations for estimating Clark unit hydrograph parameters throughout California as part of a Memorandum of Agreement with DSOD (U.S. Army Corps of Engineers, 2022).

According to Sherman, the unit hydrograph of a watershed is "…the basin outflow resulting from one unit of direct runoff generated uniformly over the drainage area at a uniform rainfall rate during a specified period of rainfall duration" (Sherman, 1932). This implies that ordinates of any hydrograph resulting from excess precipitation of unit duration would be equal to corresponding ordinates of a unit hydrograph for the same areal distribution of rainfall, multiplied by the ratio of rainfall excess values. That is, the convolution between the unit hydrograph and the excess rainfall intensity is linear. However, due to differences in areal distributions of rainfall and hydraulic reactions between large and small precipitation events, the corresponding unit hydrographs have not been found to be equal, as implied by unit hydrograph theory (Minshall, 1960; Meyersohn, 2016). These realizations must also be combined with the fact that most precipitation events used when calibrating hydrologic models are normally much less intense, which is a common characteristic of more frequent observed events, than much less frequent hypothetical events that are used to design and evaluate performance of dams and other water resources infrastructure.

In an attempt to use reasonable runoff parameters, account for non-linear response of watersheds, and address some restrictions imposed by the linearity of unit hydrograph theory, guidance has been followed within USACE for approximately 50 years requiring the use of unit hydrograph peaking factors between 1.25 and 1.5 when simulating design storms (e.g. the Probable Maximum Precipitation, PMP) within dam safety studies (U.S. Army Corps of Engineers, 1991). In essence, T_c and R values that increase the calibrated unit hydrograph peak discharge by 25- and 50-percent are used. This action shifts the resultant peak unit response at the location of interest upwards and earlier in time while maintaining the same runoff volume. These peaking factors are typically applied uniformly throughout time and space. The concept of peaking the unit response at a particular location is visualized within Figure 1.

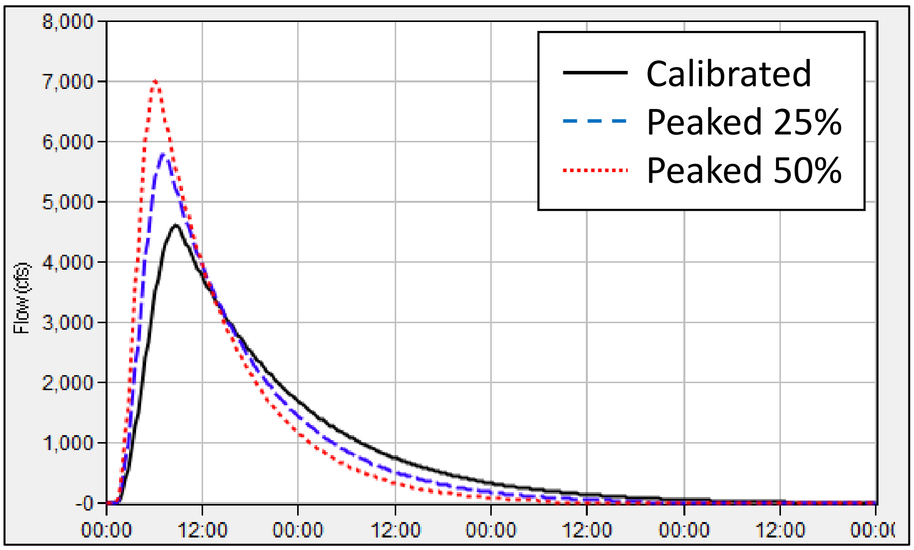

Figure 1. Peaking the Unit Response of a Watershed

However, the applicability of these rules of thumb is not thoroughly analyzed within most dam safety studies. For instance, it is unknown whether a 25% peaking factor over or under-predicts the true unit hydrograph of a watershed in response to an extreme precipitation event. Similarly, it is unknown whether a 50% peaking factor over or under-predicts the true unit hydrograph. Also, applying these peaking factors uniformly in time and space likely overpredicts the runoff response of a watershed during times of low excess precipitation rates (Bartles & Fleming, 2016).

Variable Clark Unit Hydrograph Transform

The Variable Clark unit hydrograph method can be used to avoid the aforementioned limitations by allowing both T_c and R to change as excess precipitation rates increase or decrease during a storm. Anticipated increases and decreases in translation time and/or attenuation can be simulated with excess precipitation-dependent T_c and R relationships. Excess precipitation vs. T_c and R relationships can be derived from simulations that utilize two-dimensional (2D) runoff transform methods and the resultant excess precipitation vs. T_c and R relationships can be used within the Hydrologic Engineering Center's Hydrologic Modeling System (HEC-HMS) (Bartles, 2017).

When used during extreme event simulations, this allows the unit response to vary in both a spatially and temporally appropriate manner (Bartles, 2014). For example, the unit response of impacted subbasins will only be modified during the most extreme periods of excess precipitation. Furthermore, the runoff response achieved with the Variable Clark method has been shown to achieve results that are similar to those of much more complex two-dimensional routing methods in a fraction of the computational time (Bartles & Fleming, 2016).

2D Overland Transform

Another method to estimate the outflow from a subbasin is the use of governing equations which are numerically solved. HEC's 2D engine is one such example of this approach. This 2D engine solves the St. Venant Equations using physically measurable characteristics to route water on the overland surface (U.S. Army Corps of Engineers, 2022). This engine makes use of an implicit finite volume algorithm which allows for advantages such as:

- Larger computational time steps than explicit methods,
- Improved stability and robustness over traditional finite difference and finite element techniques,
- Efficient wetting and drying of 2D cells, and
- Subcritical, supercritical, and mixed flow regimes.

Unstructured or structured computational meshes can be utilized within this engine that include triangular, square, rectangular, or even eight-sided elements. Computational cells and cell faces are pre-processed to contain detailed hydraulic property tables including elevation-volume and elevation-conveyance relationships, amongst others. This type of model is often referred to as a "high resolution subgrid model" (Casulli, 2008).

The 2D engine can be used to better recreate anticipated non-linear runoff responses when subjected to large amounts of precipitation when compared to unit hydrograph transform methods (Bartles, 2017). However, 2D overland transform methods require additional data and are more computationally intensive than unit hydrograph transform methods which may not be cost-effective in practice compared to more traditional transform methods.

Purpose and Scope

The California Department of Water Resources, Division of Safety of Dams (DSOD) requested that the USACE Hydrologic Engineering Center (HEC) derive a method to estimate variable Clark unit hydrograph parameters throughout the state of California for use within hydrologic modeling applications of extreme rainfall. Numerous studies have shown that relationships between synthetic unit hydrograph parameters and watershed characteristics can be developed and successfully used to predict parameters for independent watersheds (U.S. Army Corps of Engineers, 1982; Sabol, 1988; Holnbeck & Parrett, 1996; Melching & Marquardt, 1997; Wilkerson & Merwade, 2010).

Through the use of these equations, synthetic unit hydrograph parameters may be derived for independent watersheds as long as the watershed characteristics are hydrologically similar to the watersheds for which the relation was developed.

Hydrologic similarity includes similarity in topography, geomorphology, soil types, land cover/ land use, and climate, amongst others.

The following sections describe the procedures that were utilized to develop and test regression equations relating Clark unit hydrograph parameters and physical characteristics for watersheds in California to multiple excess precipitation rates. Additionally, the accuracy and application of the regression equations are illustrated.

Methodology

The development of the variable Clark unit hydrograph parameter regression equations used the following steps:

- Detailed analysis of rainfall and runoff data to identify suitable locations and storm events,
- Construction of a hydrologic model for each location,
- Determination of watershed characteristics using publicly available Geographic Information System (GIS) data,
- Estimation of initial hydrologic model parameters,
- Calibration of loss, baseflow, and 2D transform parameters,
- Creation of hydrographs representing 0.25, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, and 6.0 in/hr excess precipitation rates,
- Calibration of Clark unit hydrograph parameters (T_c and R) for each excess precipitation rate,
- Selection of index Clark unit hydrograph parameters,
- Derivation of dimensionless ratios of T_c and R for each excess precipitation rate relative to index parameters,
- Development of regression equations relating ratios of T_c and R for each excess precipitation rate to watershed characteristics, and
- Validation of regression equations using independent data.

Study Area and Locations

California has a land area of approximately 155,000 square miles (sq. mi.) and also contains an extremely diverse geography which ranges from the Pacific Coast in the west to the Sierra Nevada mountains in the east. In the northwestern portion of the state, coniferous forests are prevalent while the Mojave Desert can be found in the southeast. Additionally, the approximately 50-mile wide and 450-mile long Central Valley stretches across the center of the state. Similar to geography, the climate within California is also tremendously varied. A Mediterranean climate is prevalent throughout the Central Valley while moist temperate rainforests can be found in the north. Additionally, arid desert regions are widespread in the southern half of the state while snowy alpine areas can be found in the Sierra Nevada mountains.

Sixteen locations, corresponding to either U.S. Geological Survey (USGS) stream gages or USACE reservoirs, were chosen throughout California to develop variable Clark unit hydrograph parameter regression equations. Due to time and funding constraints, only sixteen locations were utilized within this study. These locations were chosen from the list of 100 locations described in U.S. Army Corps of Engineers (2022). Attempts were made to choose locations that were unimpacted by the effects of impaired flow using qualitative ratings supplied by the USGS. However, due to the widespread use of regulating structures (e.g., dams and diversions) throughout the state, some of the chosen locations were impacted by impaired flow. These effects were investigated and quantified for each location and only locations that demonstrated minimal impacts were selected for use. Drainage areas ranged from 8 to 113 sq. mi., as shown within Table 1.

A further three locations were used for validating the variable Clark unit hydrograph parameter regression equations. Details of these validation locations are contained within Table 2. The validation process is detailed within the **Regression Equation Validation** section. The positions of all locations used within this study are shown within in Figure 2.

Table 1. Locations

Name	Long	Lat	USGS Number	Drainage Area (sq. mi.)
Austin C Nr Cazadero CA	-123.07	38.51	11467200	63
Sonoma C A Agua Caliente CA	-122.49	38.32	11458500	58
Coyote C Nr Gilroy CA	-121.49	37.08	11169800	109
San Lorenzo R A Big Trees CA	-122.07	37.04	11160500	106
Miguelito C A Lompoc CA	-120.47	34.63	11134800	12
Rainbow C Nr Fallbrook CA	-117.20	33.41	11044250	10
Snow C Nr White Water CA	-116.68	33.87	10256500	11
NF Cache C A Hough Spring Nr Clearlake Oaks CA	-122.62	39.17	11451100	60
Putah C Nr Guenoc CA	-122.52	38.78	11453500	113
Bear Dam Inflow	-120.23	37.37	N/A[1]	72
Owens Dam Inflow	-120.19	37.31	N/A[1]	26
SF Tule R Nr Cholollo Campground Nr Porterville CA	-118.65	36.05	11203580	20
Marble Fork Kaweah R Ab Horse C Nr Lodgepole CA	-118.70	36.61	11206820	8
W Fk Carson Rv At Woodfords CA	-119.83	38.77	10310000	65
Truckee R Nr Truckee CA	-120.21	39.30	10338000	46[2]
Big Rock C Nr Valyermo CA	-117.84	34.42	10263500	23

[1]*Computed inflow provided by USACE*
[2]*Uncontrolled drainage area below Lake Tahoe Outlet*

Table 2. Validation Locations

Name	Long	Lat	USGS Number	Drainage Area (sq. mi.)
EF Russian R Nr Calpella CA	-123.13	39.25	11461500	92
Arroyo Seco Nr Pasadena CA	-118.18	34.22	11098000	16
Elder C Nr Paskenta CA	-122.51	40.02	11379500	92

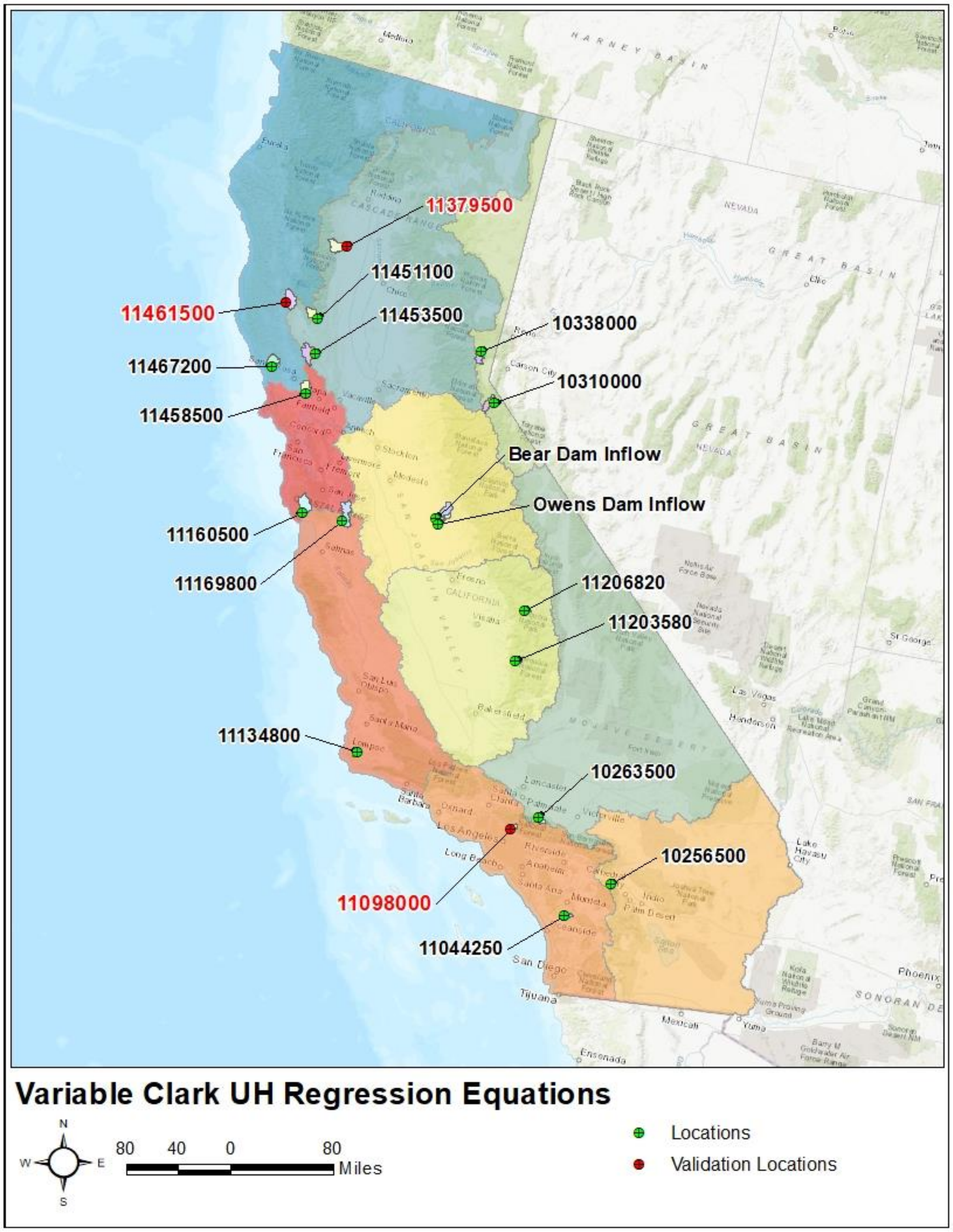

Figure 2. Locations

Data Compilation

Multiple sources of data were collected/reviewed for use within this modeling effort including geographic and climatic information, field observations, previous reports, and water control manuals, amongst others. These sources included:

- Stream gages operated by the U.S. Geological Survey (USGS)
  - [USGS National Water Information System]

- Observed reservoir data at USACE projects were downloaded from Corps Water Management System (CWMS) servers at the Sacramento District (SPK)
- Watershed boundaries and streams
    - USGS National Hydrography
- 10-meter digital elevation models (DEM)
    - USGS National Map Viewer
- Land use classifications
    - Multi-Resolution Land Use Consortium (MRLC)
- Multi-Radar Multi-Sensor (MRMS) Quantitative Precipitation Estimate (QPE)
    - IA State Mesonet (2015-present)
- Real-time Mesoscale Analysis (RTMA) Precipitation and Temperature
    - IA State Mesonet (2012-present)
- Parameter-elevation Regressions on Independent Slopes Model (PRISM)
    - PRISM Climate Group
- Next-generation Radar (NEXRAD) QPE
    - NCEI Orders
- National Oceanic and Atmospheric Administration (NOAA) National Centers for Environmental Prediction (NCEP) Stage IV Precipitation
    - NCAR / EOL
- Analysis of Record for Calibration (AORC) Precipitation and Temperature
    - FTP
- NOAA's Snow Data Assimilation System (SNODAS)
    - SNODAS
- University of Arizona (UofA) Snow
    - UofA Snow

The gridded boundary condition data source that was used for each modeling domain depended upon the age of the event used for model calibration. For instance, MRMS data was used for events that occurred after 2015. When large under or overpredictions of precipitation were found relative to runoff observations, the Normalizer tool within HEC-HMS was used to create normalized precipitation grids. In all cases where normalization was required, PRISM data was used to correct the sub-daily precipitation. PRISM data has been shown to provide accurate estimations of daily precipitation accumulation within ungaged locations, such as mountainous locations or rural areas (Daly, Slater, Roberti, Laseter, & Swift, 2017). The Gridded Data Import tool within HEC-HMS was used to import the gridded precipitation, temperature, and snow data. During the import process, each data set was reprojected to a common coordinate reference system, resampled to a 2 km x 2 km grid size, and clipped to the boundary of the modeling domain.

HEC-HMS Model Development

HEC-HMS was used to simulate the precipitation-runoff processes within each modeling domain. HEC-HMS has been successfully used to solve a wide array of possible hydrologic problems including large river basin water supply, extreme flood hydrology, and small urban watershed runoff, amongst other uses (U.S. Army Corps of Engineers, 2021). Previously built HEC-HMS models were used as the starting points for each modeling domain (U.S. Army Corps of Engineers, 2022).

The previously mentioned NED 10-meter DEMs were used as the basis for all watershed and stream delineations and physical parameter estimations. A 2D mesh was created within HEC-RAS for each modeling domain. Breaklines were used to align cell faces with prominent topographic features like roadways, embankments, and stream centerlines. Computation points were added in order to create approximately 1000 to 5000 cells, depending upon the size and complexity, within each modeling domain. To create 2D meshes with variable computation point spacing, improve accuracy, and reduce computation times, Mesh Refinement Regions were also used within some modeling domains. Following the creation of each 2D mesh within HEC-RAS, the resultant information was imported to HEC-HMS.

The following modeling methods were used within all models:

- Deficit and Constant Loss,
- 2D Diffusion Wave Transform,
- Linear Reservoir Baseflow, and
- Gridded Temperature Index Snowmelt (when necessary).

Watershed Characteristics

Multiple watershed characteristics were evaluated for the contributing drainage area at each location. These characteristics included:

- Drainage Area (DA). This parameter is in sq. mi.
- Longest Flowpath Length (L), which is the length along the longest watercourse from the watershed outlet to the upper limit of the watershed boundary. This parameter is in miles.
- Centroidal Flowpath Length (L_{ca}), which is a subset of L. L_{ca} is the length along the longest watercourse from the watershed outlet to a point on the stream nearest the watershed centroid. This parameter is in miles.
- 10-85 Flowpath Length (L_{10-85}), which is also a subset of L. Measuring from the outlet in the upstream direction, L_{10-85} is the length along the longest watercourse beginning at a point representing ten percent of L and extending to a point representing eighty-five percent of L. This parameter is in miles.
- 10-85 Flowpath Slope (S_{10-85}), which is the average slope of L_{10-85} and is dimensionless.
- Basin Slope (S), which represents the average slope of the entire subbasin and is dimensionless.
- Basin Relief (BR), which is calculated as the difference between the highest elevation on the drainage divide and the elevation of the outlet point of the subbasin. This parameter is in feet.
- Relief Ratio (RR), which is computed using the following equation and is dimensionless:

$$RR = \frac{BR}{L} \tag{1}$$

- Elongation Ratio (ER), which is a dimensionless ratio used to categorize the general shape of a watershed by comparing the diameter of a circle with the same area as the subbasin and L. ER was computed using the following equation and is dimensionless:

$$ER = \left(\frac{\sqrt{DA}}{L}\right) * \left(\frac{2}{\sqrt{\pi}}\right) \tag{2}$$

- Drainage Density (DD), which is a metric used to describe the efficiency in which a subbasin is drained by stream channels. DD was computed using the following equation and reported in $\frac{mi}{sq\ mi}$:

$$DD = \frac{Stream\ Miles}{DA} \tag{3}$$

  where Stream Miles is derived from the NHD.
- LSqrtS, which is computed using the following equation and reported in miles:

$$LSqrtS = \frac{L}{\sqrt{S_{10-85}}} \tag{4}$$

- Basin shape factor (BSF), which is computed using the following equation and reported in sq. mi. (U.S. Army Corps of Engineers, 1982):

$$BSF = \frac{L * L_{ca}}{\sqrt{S_{10-85}}} \tag{5}$$

- LSqrtDAS, which is computed using the following equation and is in sq. mi.:

$$LSqrtDAS = L * \sqrt{\frac{DA}{S_{10-85}}} \tag{6}$$

HEC-HMS Historical Event Model Calibration

In order to develop 2D simulation outputs that can be used to estimate variable Clark unit hydrograph parameters for multiple excess precipitation rates as needed within this analysis, initial process parameters and inputs were “ground-truthed” to better reflect watershed conditions during historical precipitation events.

It is anticipated that DSOD will utilize the regression equations developed within this study to estimate variable Clark unit hydrograph parameters to simulate extreme events within dam safety studies. As such, large and significant storm events were selected for use during model calibration. However, observed data availability, reliability, and ease of use were also considered. Events that were used for each location are shown within Table 3. Only one event per location was used due to time and funding constraints.

Table 3. Calibration Events

Location	**Event**	**Observed Peak Flow (cfs)**	**Observed Unit Peak (cfs/sq. mi.)**
Austin C near Cazadero CA	Feb 2019	21300	338.1
Sonoma C at Agua Caliente CA	Dec2005 - Jan2006	20300	350
Coyote C near Gilroy CA	Jan 2017	11500	105.5
San Lorenzo R at Big Trees CA	Feb 2017	19000	179.2

Location	Event	Observed Peak Flow (cfs)	Observed Unit Peak (cfs/sq. mi.)
Miguelito C at Lompoc CA	Feb 1998	2660	221.7
Rainbow C near Fallbrook CA	Dec 2010	4010	401
Snow C near White Water CA	Feb 2019	3170	288.2
NF Cache C at Hough Spring near Clearlake Oaks CA	Feb 2019	10100	168.3
Putah C near Guenoc CA	Jan 1997	24000	212.4
Bear Dam Inflow	Mar 2018	25500	354
Owens Dam Inflow	Mar 2018	10500	403.8
SF Tule R near Cholollo Campground near Porterville CA	Feb 2017	1510	75.5
Marble Fork Kaweah R Ab Horse C near Lodgepole CA	Jul 2015	997	124.6
W Fk Carson Rv at Woodfords, CA	Jan 1997	8100	124.6
Truckee R near Truckee CA	Jan 1997	11900	258.7[1]
Big Rock C near Valyermo CA	Jan 2005	2550	110.9

[1]*Unit peak discharge represents uncontrolled drainage area below Lake Tahoe Outlet*

Model performance was evaluated by comparing computed results against observed results at each location. Model parameters were altered to minimize the differences between computed and observed hydrograph shape, peak flow rate, and discharge volume. Differences less than or equal to 15 percent between computed and observed peak flow rates and flow volumes were desired. Additionally, when adequate observed data was available (at a minimum, daily average streamflow), summary statistics were used to quantify model performance (Moriasi, et al., 2007). Statistical metrics included Nash-Sutcliffe Efficiency (NSE), Ratio of the Root Mean Square Error to the Standard Deviation (RSR), and Percent Bias (PBIAS) (U.S. Army Corps of Engineers, 2021). Observed data that was available for use varied by location and flood event. When available, 15-, 30-, or 60-minute observed streamflow was used. When that data was not available, daily average and instantaneous peak streamflow was used. Table 4 contains statistical metrics for each location. Table 5 shows the results of the calibrated Clack unit hydrograph parameters to the 16 modeling domains.

Table 4. Calibration Statistical Metrics

Location	RSR	NSE	PBIAS
Austin C near Cazadero CA	0.30 (Very Good)	0.92 (Very Good)	-14.51 (Very Good)
Sonoma C at Agua Caliente CA	0.40 (Very Good)	0.81 (Very Good)	-6.79 (Very Good)
Coyote C near Gilroy CA	0.40 (Very Good)	0.84 (Very Good)	-0.93 (Very Good)

Location	RSR	NSE	PBIAS
San Lorenzo R at Big Trees CA	0.20 (Very Good)	0.96 (Very Good)	-1.62 (Very Good)
Miguelito C at Lompoc CA	0.70 (Satisfactory)	0.50 (Satisfactory)	66.67 (Unsatisfactory)
Rainbow C near Fallbrook CA	0.50 (Very Good)	0.79 (Very Good)	26.95 (Satisfactory)
Snow C near White Water CA	0.50 (Very Good)	0.71 (Very Good)	-10.27 (Very Good)
NF Cache C at Hough Spring near Clearlake Oaks CA	0.30 (Very Good)	0.88 (Very Good)	-2.54 (Very Good)
Putah C near Guenoc CA	0.20 (Very Good)	0.97 (Very Good)	2.55 (Very Good)
Bear Dam Inflow	0.40 (Very Good)	0.81 (Very Good)	-6.93 (Very Good)
Owens Dam Inflow	0.30 (Very Good)	0.90 (Very Good)	10.98 (Very Good)
SF Tule R near Chololllo Campground near Porterville CA	0.40 (Very Good)	0.88 (Very Good)	2.03 (Very Good)
Marble Fork Kaweah R Ab Horse C near Lodgepole CA	0.70 (Satisfactory)	0.57 (Satisfactory)	4.77 (Very Good)
W Fk Carson Rv at Woodfords, CA	0.50 (Very Good)	0.75 (Very Good)	0.87 (Very Good)
Truckee R near Truckee CA	0.39 (Very Good)	0.84 (Very Good)	8.53 (Very Good)
Big Rock C near Valyermo CA	0.49 (Very Good)	0.76 (Very Good)	-1.65 (Very Good)

Table 5. Calibrated Clark Unit Hydrograph Parameters

Location	Maximum Excess Precipitation Rate (in/hr)	Tc (hr)	R (hr)
Austin C near Cazadero CA	0.48	3	5
Sonoma C at Agua Caliente CA	0.68	1.6	4.3
Coyote C near Gilroy CA	0.24	7	3.5
San Lorenzo R at Big Trees CA	0.52	5.75	3.5
Miguelito C at Lompoc CA	0.56	3.2	1.5
Rainbow C near Fallbrook CA	0.56	0.5	1
Snow C near White Water CA	0.64	4	1.5
NF Cache C at Hough Spring near Clearlake Oaks CA	0.2	1.5	7.5
Putah C near Guenoc CA	0.52	4.5	5
Bear Dam Inflow	1.16	2.5	1.5
Owens Dam Inflow	1.16	2.5	0.5
SF Tule R near Chololllo Campground near Porterville CA	0.2	4	5

Location	Maximum Excess Precipitation Rate (in/hr)	Tc (hr)	R (hr)
Marble Fork Kaweah R Ab Horse C near Lodgepole CA	0.12	0.75	1.5
W Fk Carson Rv at Woodfords, CA	0.21	4	8
Truckee R near Truckee CA[1]	0.68	7	8
Big Rock C near Valyermo CA	0.2	2.5	2.5

[1]Parameters represent uncontrolled drainage area below Lake Tahoe Outlet

HEC-HMS Hypothetical Event Model Calibration

Following the calibration of each historical event HEC-HMS model, a single pulse of 0.25-, 0.5-, 1.0-, 2.0-, 3.0-, 4.0-, 5.0-, and 6.0-inches of excess precipitation over one hour was applied to each modeling domain and routed using the calibrated 2D diffusion wave transform parameters. No losses or baseflow processes were included within these simulations. The resultant runoff hydrographs of these 2D hypothetical event simulations were then used to calibrate Clark unit hydrograph parameters for each excess precipitation rate.

Model performance for each excess precipitation rate was evaluated by comparing results computed using the Clark unit hydrograph transform method against results computed using the 2D diffusion wave transform method at each location. Clark unit hydrograph parameters were systematically modified to minimize the differences when compared against the 2D diffusion wave transform results. For simplicity, the default time-area histogram included within the HEC-HMS Clark unit hydrograph transform was used, without modification, within all modeling domains. Differences in time of peak and peak flow rates less than 1 hour and 3 percent, respectively, between the Clark unit hydrograph and 2D diffusion wave results were desired. Figure 3 provides an example of Clark unit hydrograph parameters being modified to adequately approximate the 2D diffusion wave transform results.

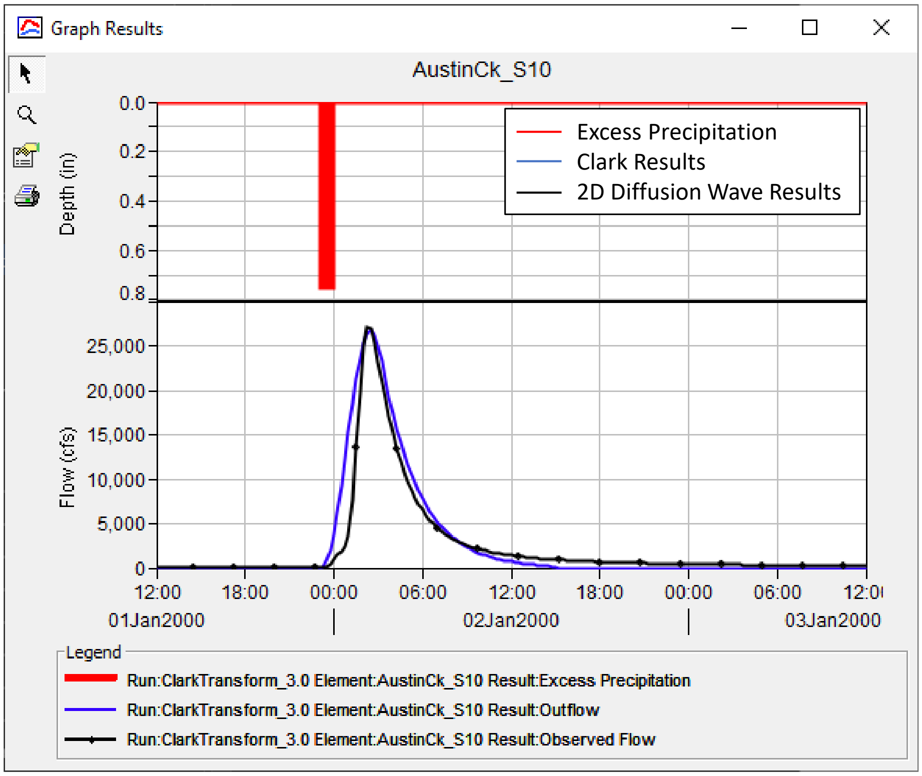

Figure 3. Example of Calibrating Clark Unit Hydrograph Parameters to Match 2D Diffusion Wave Results for 3 in/hr Excess Precipitation

Following model calibration, a second quality control review was conducted by HEC personnel. During this review, parameter modifications were investigated in addition to model performance. Efforts were made to ensure that consistent model construction, parameterizations, and calibration techniques were applied. When necessary, comments were addressed by modifying model parameters and/or recomputing new statistical metrics.

Selection of Index Clark Unit Hydrograph Parameters

Clark unit hydrograph parameters vary due to multiple factors including drainage area, watershed shape, and storm event, amongst others. In order to develop regression equations that predict how Clark unit hydrograph parameters change with excess precipitation rate throughout the state of California, which includes diverse geography, watershed sizes, and climatology, dimensionless ratios of T_c and R with respect to index values were developed. The index excess precipitation is defined as the maximum excess precipitation rate realized during an historical event simulation. The calibrated values shown within Table 5 were selected as the index parameters for each of the 16 previously mentioned modeling domains. Dimensionless ratios of T_c and R for each excess precipitation rate relative to the index parameters were then derived for each modeling domain by calculating the quotient of the calibrated parameters and the index T_c and R values, as shown in the following equations.

$$T_c, ratio = \frac{T_c}{T_c, index} \quad (7)$$

$$R, ratio = \frac{R}{R, index} \quad (8)$$

An example of the resultant dimensionless relationships for Austin Creek is shown in Figure 4.

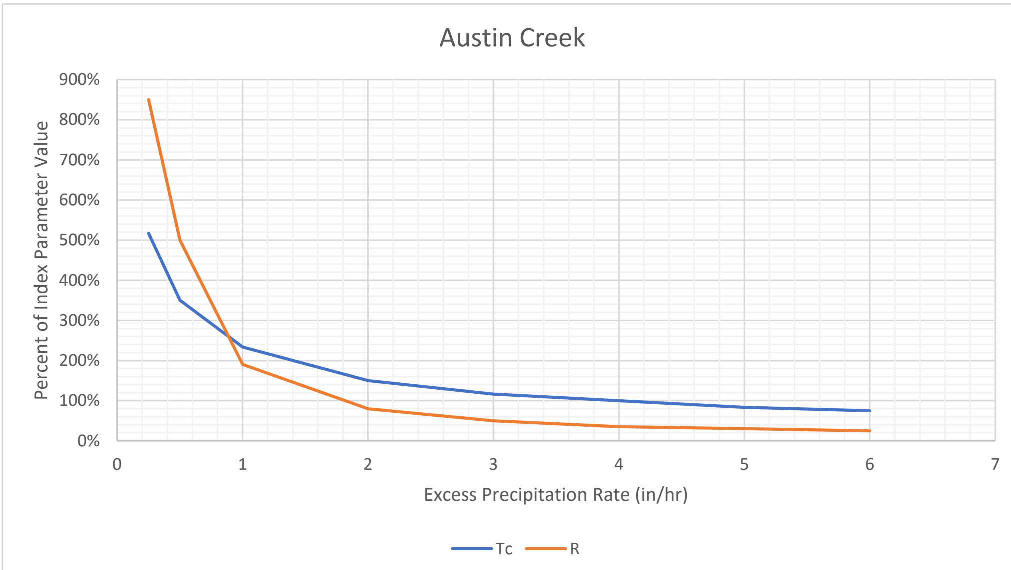

Figure 4. Example of Dimensionless Ratios of T_c and R Relative to Index Parameters for Austin Creek

Regression Equation Development

The R Statistical Language (R Core Team, 2021) was used to perform data analysis and develop regression equations relating the ratios of Clark unit hydrograph parameters to index values and physical characteristics for each of the aforementioned excess precipitation rates.

Approach

Two multiple linear regression models were considered within this study. The first model was of the form:

$$Y = a + b_1 x_1 + b_2 x_2 + \cdots + b_n x_n \quad (9)$$

where Y is the dependent variable (e.g., Ratio of $T_{c,index}$ or Ratio of $R_{,index}$); a is a regression constant; x_1, x_2, …, x_n are independent variables (e.g., watershed characteristics); and b_1, b_2, …, b_n are unknown coefficients. The second model was of the form:

$$\log_{10} Y = a + b_1 \log_{10} x_1 + b_2 \log_{10} x_2 + \cdots + b_n \log_{10} x_n \quad (10)$$

The regression constant and unknown coefficients were determined utilizing the method of least squares. When using multiple linear regression with several watershed characteristics, some of the proposed characteristics may have little to no effect on the dependent variable of interest. When this occurs, these characteristics were removed from consideration (U.S. Army Corps of Engineers, 1982). In order to simultaneously evaluate the usefulness of the regression models and avoid overfitting, an adjusted coefficient of determination (adjusted R^2) was used. Adjusted R^2 ranges between -∞ and 1, where adjusted $R^2 = 1$ is optimal. This metric increases when a new term improves the regression model more than would be expected by chance. Adjusted R^2 is computed using the following equation:

$$Adjusted\ R^2 = 1 - (1 - R^2)\frac{n-1}{n-p-1} \tag{11}$$

where p is the total number of explanatory (independent) variables, n is the sample size, and R^2 is the "classical" coefficient of determination. t-statistic tests and their associated p-values were used to determine whether there was a statistically significant relationship between an independent variable and the dependent variable of interest, that is whether or not the unknown coefficient(s) within Equation (11) are significantly different from zero. Variables with p-values ≤ 0.05 were desired. However, this criterion was not the only determining factor when evaluating explanatory variables.

Stepwise selection techniques were employed with Adjusted R^2 and p-values to select the most performant watershed characteristics. Scatterplots relating T_c and R to the watershed characteristics were developed for each of the eight previously mentioned excess precipitation rates using the ggpairs() function from the GGally package. The resultant plots and information were used to make informed decisions as to which variables were most impactful.

The watershed characteristics which were previously mentioned were considered to be error free because of the computational accuracy of using remote sensing and GIS datasets. However, the ratios of T_c and R relative to the index parameters for each modeling domain were not considered to be error free. Errors within these parameters could arise from numerous sources including, but not limited to, boundary condition information, modeling techniques, and observed data availability. Clark unit hydrograph parameter error was included by including weights corresponding to the data availability. Locations with 15-, 30-, or 60-min stream streamflow data were given a weight of 1.5 while locations with daily average and instantaneous peak streamflow data were given a weight of 1.0. This implies that results calibrated using 15-, 30-, or 60-min streamflow measurements were given 1.5 times the weight of those which were derived using daily average and instantaneous peak streamflow data. These weights were carried forward to the variable Clark unit hydrograph parameters which were derived using the 2D diffusion wave results.

Results

The various regression models were examined using several diagnostic tests to ascertain whether the models obeyed general assumptions of multiple linear regression. These diagnostic tests included checking whether:

1. A linear relationship exists within the model,
2. Residuals (or errors) are evenly distributed over the range of predicted values,
3. Residuals are normally distributed, and

4. One or more residuals do not exert excessive leverage on the model.

Regression equations were developed using all available data. Region-specific equations were not developed. This was done to increase the sample size, reduce uncertainty in the results, and make the regression equations easier to apply. Additionally, the same predictive variables were used for each excess precipitation rate to ease the use of the regression equations and mitigate the potential for sharp inflection points in the output or increasing ratios of T_c and R as excess precipitation rates increase for reasonable parameter values. The use of log10(DA) and log10($S_{10\text{-}85}$) was found to produce acceptable Adjusted R^2 values for each excess precipitation rate when predicting ratios of T_c relative to an index parameter. Similarly, the use of L_{ca}, BR, and RR was found to produce acceptable Adjusted R^2 values for each excess precipitation rate when predicting ratios of R relative to an index parameter. A summary of the regression equations for each excess precipitation rate is presented within Table 6.

Table 6. Summary of Regression Equations

Excess Precipitation Rate (in/hr)	Equation[1]
0.25	$\log_{10}T_{c,ratio} = 0.04082 - 0.35461 * \log_{10}DA - 0.53746 * \log_{10}S_{10-85}$
	$R_{ratio} = -1.01528 + 1.7684 * L_{ca} - 0.00316 * BR + 120.4306 * RR$
0.5	$\log_{10}T_{c,ratio} = -0.09866 - 0.42538 * \log_{10}DA - 0.5683 * \log_{10}S_{10-85}$
	$R_{ratio} = -1.40573 + 1.2471 * L_{ca} - 0.00208 * BR + 78.6917 * RR$
1.0	$\log_{10}T_{c,ratio} = -0.2412 - 0.5586 * \log_{10}DA - 0.6837 * \log_{10}S_{10-85}$
	$R_{ratio} = -0.26226 + 0.6196 * L_{ca} - 0.00118 * BR + 38.7834 * RR$
2.0	$\log_{10}T_{c,ratio} = -0.381 - 0.5353 * \log_{10}DA - 0.6581 * \log_{10}S_{10-85}$
	$R_{ratio} = 0.507 + 0.2515 * L_{ca} - 0.00063 * BR + 16.721 * RR$
3.0	$\log_{10}T_{c,ratio} = -0.4809 - 0.5387 * \log_{10}DA - 0.6703 * \log_{10}S_{10-85}$
	$R_{ratio} = 0.489 + 0.1709 * L_{ca} - 0.00046 * BR + 11.3057 * RR$
4.0	$\log_{10}T_{c,ratio} = -0.5446 - 0.5107 * \log_{10}DA - 0.6573 * \log_{10}S_{10-85}$
	$R_{ratio} = 0.4668 + 0.13062 * L_{ca} - 0.00037 * BR + 8.4999 * RR$
5.0	$\log_{10}T_{c,ratio} = -0.5425 - 0.5204 * \log_{10}DA - 0.6457 * \log_{10}S_{10-85}$

Excess Precipitation Rate (in/hr)	Equation[1]
	$R_{ratio} = 0.4814 + 0.10512 * L_{ca} - 0.00033 * BR + 7.2064 * RR$
6.0	$\log_{10} T_{c,ratio} = -0.5408 - 0.5403 * \log_{10} DA - 0.6452 * \log_{10} S_{10-85}$ $R_{ratio} = 0.5215 + 0.0824 * L_{ca} - 0.00029 * BR + 5.9854 * RR$

[1]DA = drainage area (sq mi), $S_{10\text{-}85}$ = 10-85 Flowpath Slope (dimensionless), L_{ca} = Centroidal Flowpath Length (mi), BR = Basin Relief (feet), and RR = Relief Ratio (dimensionless)

Uncertainty

The regression equations presented within the previous sections represent the most likely T_c or R value for a given location. However, the regression equations contain uncertainty which cannot be completely eliminated. This uncertainty stems primarily from the difficulty in recreating the results obtained from the 2D diffusion wave transform using only two parameters (T_c and R). To quantify the amount of uncertainty present when estimating a T_c or R for a specific location, prediction intervals can be quantified using the following equation:

$$y^* \pm t_{\alpha/2,n-2)}\sqrt{MSE + [SE(y^*)]^2} \quad (12)$$

where y^* = predicted value (T_c or R), $t_{\alpha/2,n-2}$ = t distribution critical value, n = number of observations, and $MSE + [SE(y^*)]^2$ = the standard error of the prediction. Prediction intervals should be used when estimating the uncertainty around a yet-to-be-observed data point since they incorporate both the model parameter uncertainty (e.g., error around the population mean at the input value) in addition to the residual uncertainty. When using weighted multiple linear regression, as is the case within this study, the estimation of the standard error of the prediction is complex.

Uncertainty about any predicted ratio of T_c or R relative to an index parameter is assumed to be normally distributed. As such, prediction intervals derived using the aforementioned equations and information can be used to estimate the amount of uncertainty in hydrologic model outputs (e.g., runoff volume, peak discharge, peak reservoir elevation, etc.) that is due to use of variable Clark unit hydrograph parameters predicted using the regression equations.

Regression Equation Validation

Following the development of regression equations for each excess precipitation rate, the accuracy of the regression equations was quantified using a validation process. In order to validate the regression equations, three independent locations were selected that were not included in the 16 modeling domains selected for the development of the regression equations. Following selection of these locations, watershed characteristics were extracted for each modeling domain and variable Clark parameters were estimated using the aforementioned regression equations. Next, hydrologic models were constructed for each location using the same processes that were previously described. 2D diffusion wave parameters were calibrated using one historical event for each location. Following calibration, multiple hypothetical events were simulated for each location using the calibrated 2D diffusion wave transform parameters. Finally, the 2D diffusion wave results were compared against those obtained through the use of the predicted variable Clark unit hydrograph parameters. Three locations were used for validation, as shown within Table 2. The positions of these locations are shown within in Figure 2.

2D Diffusion Wave Calibration

In order to develop 2D diffusion wave results that could be used for comparison to the predicted values obtained through the Variable Clark unit hydrograph parameter regression equations, initial parameter estimates for each validation location were subjected to a model calibration processes. Events that were used to calibrate for each location are shown within Table 7. Only one event per location was used. 15-, 30-, or 60-minute streamflow data was available for each location.

Table 7. Validation Events used to Calibrate 2D Diffusion Wave Parameters

Location	Event	Observed Peak Flow (cfs)	Observed Unit Peak (cfs/sq. mi.)
EF Russian R Nr Calpella CA	Dec2005 - Jan2006	14600	158.7
Arroyo Seco Nr Pasadena CA	Jan2010	4230	264.4
Elder C near Paskenta CA	Feb 2019	12700	138

Variable Clark Parameter Estimation

The maximum excess precipitation rate, calibrated T_c, and calibrated R values shown in Table 8 were selected as the index parameters for each location.

Table 8. Index Clark Unit Hydrograph Parameters for Each Validation Location

Location	Index Excess Precipitation Rate (in/hr)	Index T_c (hr)	Index R (hr)
EF Russian R Nr Calpella CA	0.4	5	8
Arroyo Seco Nr Pasadena CA	0.66	1.6	0.75
Elder C near Paskenta CA	0.2	4	3

The Variable Clark unit hydrograph parameter regression equations were then applied to each location in order to estimate ratios of T_c and R for excess precipitation rates of 0.25, 0.5, 1, 2, 3, 4, 5, and 6 in/hr. Each of the aforementioned excess precipitation rates were divided by the index excess precipitation for each location to derive % of Excess $\text{Precipitation}_{,\text{index}}$. The index excess precipitation is defined as the maximum excess precipitation rate realized during an historical event simulation. Two additional points representing (0, Index T_c, Index R) and (Index Excess Precipitation Rate, Index T_c, Index R) were added to the relationships for each location. Finally, ratios of T_c and R greater than 100% were set equal to 100% and any minor inflection points were smoothed. This ensures that no values greater than the index T_c and R are used and T_c and R decrease as excess precipitation rate increases. An example of these final relationships for Arroyo Seco is shown in Table 9.

Table 9. Final Ratios of Excess Precipitation, T_c, and R Relative to Index Parameters for Arroyo Seco

% of Excess Precipitation$_{,index}$	**% of $T_{c,index}$**	**% of $R_{,index}$**
0	100	100
37.9	100	100
75.8	100	100
100	100	100
151.5	100	100
303.0	72.8	49.3
454.5	59.5	30.5
606.1	53.4	24.9
757.6	50.3	19.1
909.1	47.8	16.7

Results

Following calibration of the 2D diffusion wave transform parameters, multiple hypothetical events were simulated for each validation location and the results were used to ascertain the accuracy of the predicted variable Clark parameters. First, results from Design Flood simulations (e.g., Probable Maximum Flood, PMF; 1/1000 Annual Exceedance Probability, AEP) were compared at each location. These types of simulations are commonly used within dam safety studies undertaken by DSOD. Additionally, predicted variable Clark unit hydrograph parameters were altered to minimize differences when compared against 2D diffusion wave transform results for excess precipitation rates of 0.25, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, and 6.0 in/hr with no losses or baseflow contributions. The resultant calibrated variable Clark unit hydrograph parameters were then compared against the predicted variable Clark unit hydrograph parameters.

Within the following paragraphs, “predicted” values were obtained from the variable Clark unit hydrograph parameter regression equations while “calibrated” values were obtained through the use of the 2D diffusion wave transform. "Training” data refers to the results for the 16 locations detailed in Table 1 while “Validation” data refers to the three locations detailed within Table 2.

An approximation of the PMF was simulated for the EF Russian River location using input from a recent dam safety study (U.S. Army Corps of Engineers, 2009). For reference, the 72-hr duration Probable Maximum Precipitation (PMP) at this location is approximately 30.5 inches. The peak runoff response obtained through the use of the variable Clark unit hydrograph parameter regression equations was slightly larger (approximately 8.24%) than the 2D diffusion wave transform results for the PMF event at this location, as shown within Figure 5. However, the variable Clark unit hydrograph results closely matched the 2D diffusion wave transform results in magnitude, time of peak discharge, and hydrograph shape. As such, these results are acceptable.

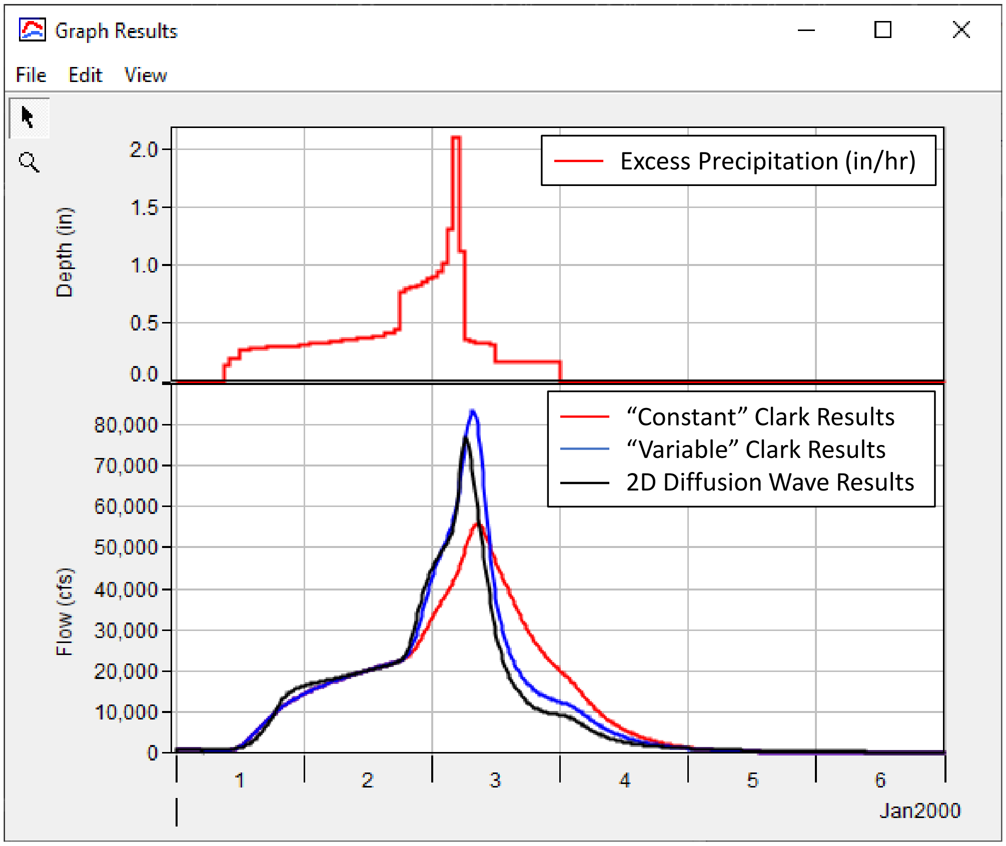

Figure 5. EF Russian River PMF Results

An approximation of the 24-hr duration 1/1000 AEP event was simulated for the Arroyo Seco location using input from Atlas 14, Volume 6 (National Weather Service, 2014). For reference, the 24-hr duration 1/1000 AEP rainfall depth at this location is approximately 14.6 inches. The Jan 2010 temporal pattern was used to distribute this precipitation in time. The peak runoff response obtained through the use of the variable Clark unit hydrograph parameters closely matches the hydrograph from the 2D diffusion wave transform at this location, as shown within Figure 6. The hydrograph from the constant Clark parameters predicts a much lower peak discharge occurring at a later time. It is apparent from Figure 6 the benefits of using a variable Clack parameter approach to hydrologic routing. These validation results are acceptable given the magnitude of the design flood simulation and the excellent statistical metrics.

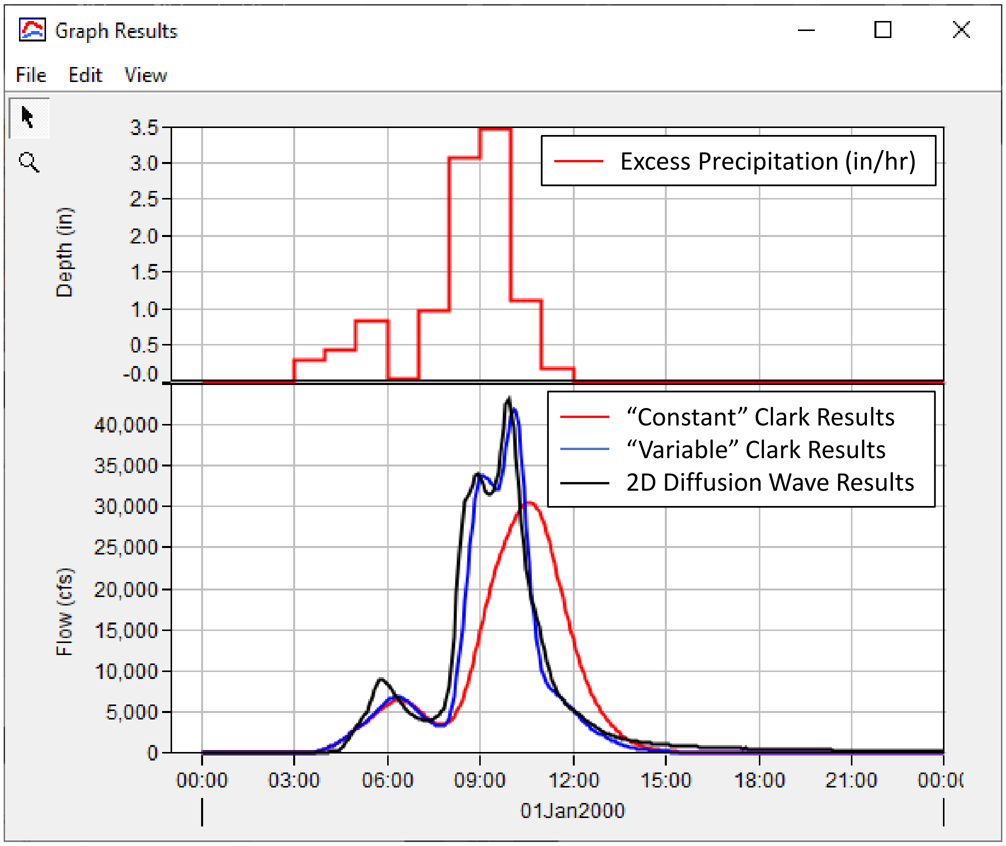

Figure 6. Arroyo Seco 1/1000 AEP Event Results

An approximation of the 72-hr duration 1/1000 AEP event was simulated for the Elder Creek location using input Atlas 14, Volume 6 (National Weather Service, 2014). For reference, the 72-hr duration 1/1000 AEP rainfall depth at this location is approximately 12.6 inches. The Feb 2019 temporal pattern was used to distribute this precipitation in time. The peak runoff response obtained through the use of the variable Clark unit hydrograph parameter regression equations was found to be slightly less (approximately 11%) than the 2D diffusion wave transform results for the 1/1000 AEP event at this location, as shown within Figure 7. However, the validation results are well within the bounds of acceptability.

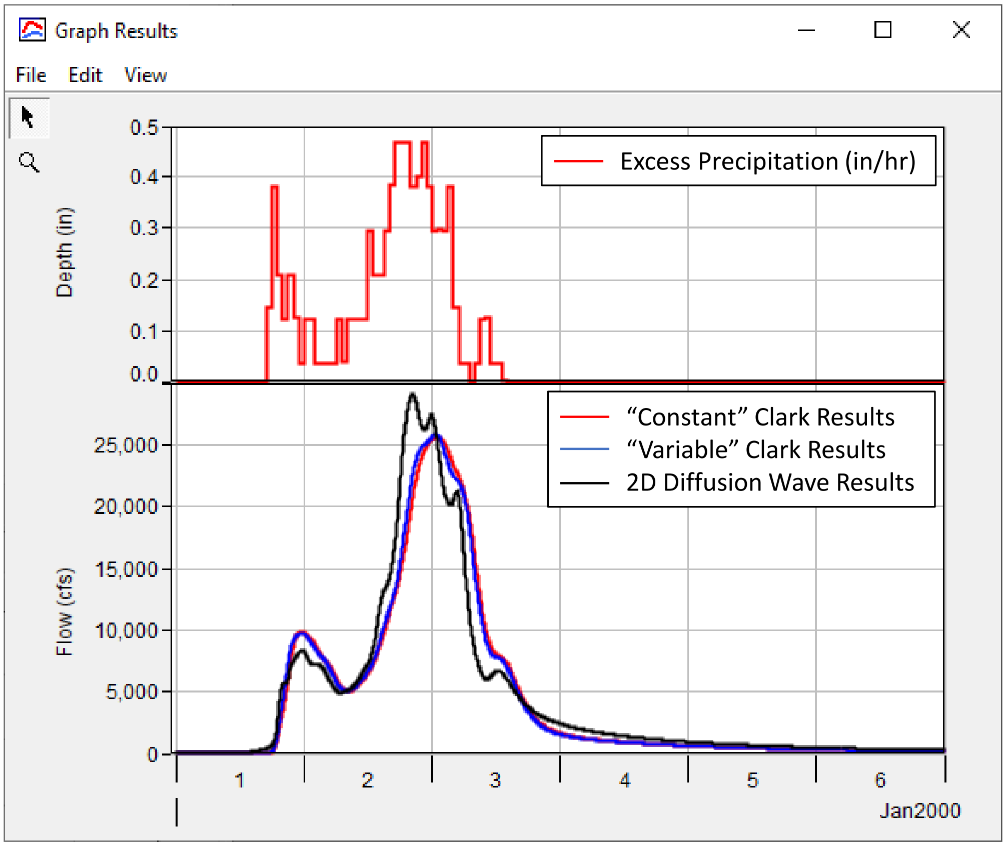

Figure 7. Elder Creek 1/1000 AEP Event Results

The regression equations developed to estimate ratios of T_c and R relative to index parameters for varying excess precipitation rates were shown to successfully validate using calibrated results. As an example, the T_c and R validation results for an excess precipitation rate of 2 in/hr are shown in Figure 8 and Figure 9, respectively.

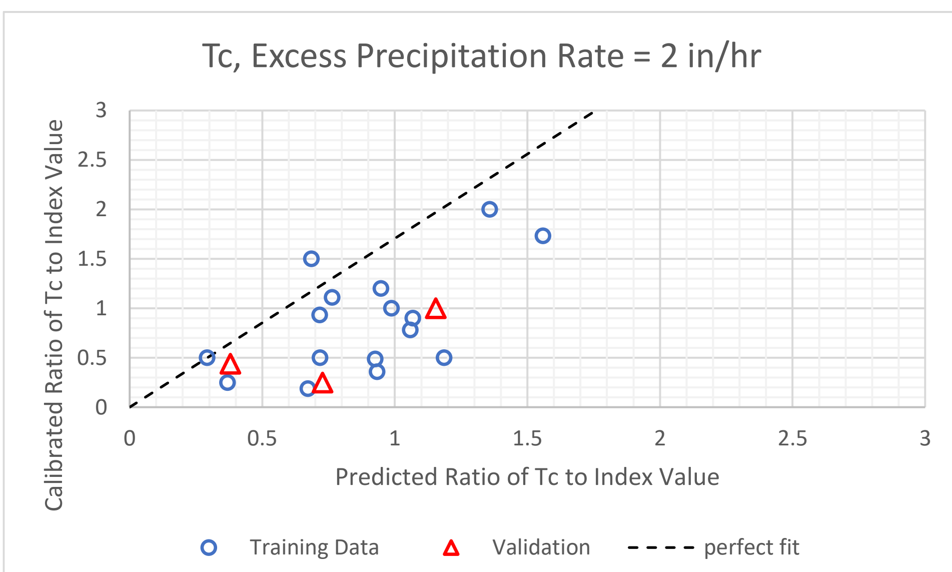

Figure 8. Excess Precipitation Rate = 2 in/hr: Ratio of $T_{c,index}$ Validation Results

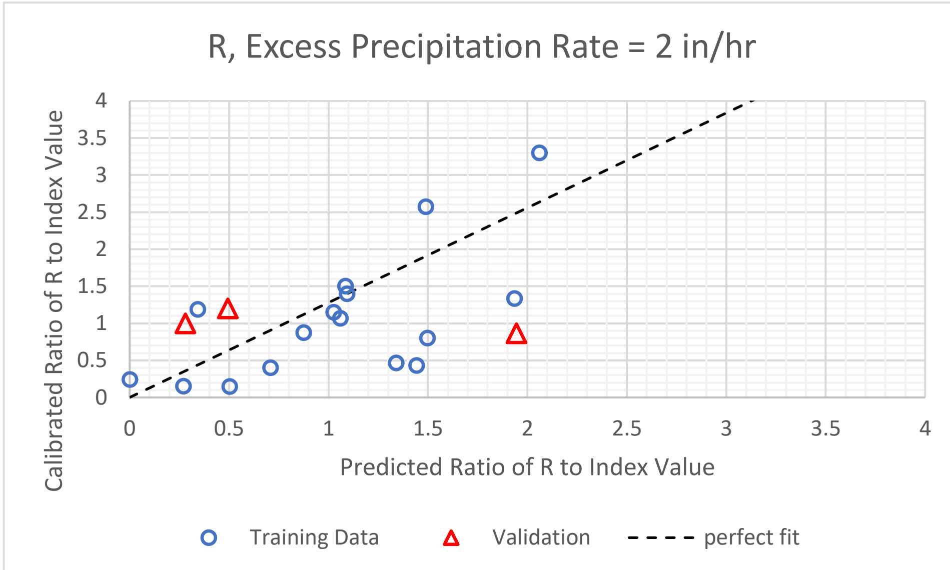

Figure 9. Excess Precipitation Rate = 2 in/hr: Ratio of $R_{,index}$ Validation Results

Conclusions

The regression equations developed as part of this effort can be used to predict variable Clark unit hydrograph parameters throughout the entire state of California. These equations relate physically measurable watershed characteristics to T_c and R for multiple excess precipitation rates. Clark unit hydrograph parameters should be initially estimated using calibration or any available regional regression equations (U.S. Army Corps of Engineers, 2022). When observed runoff data is available, these initial estimates should be subjected to a model calibration process that compares computed outputs against observed data. Parameters should be modified in order to achieve an adequate fit. Following model calibration, parameters should be tested through a model validation process where computed results, without any further parameter modifications, are used to compute outputs which are compared against observed data for independent events that were not considered during model calibration.

When observed runoff data is available, index Clark unit hydrograph parameters should then be equated to the calibrated parameters. However, when observed runoff data is not available, Clark unit hydrograph parameters estimated using regression equations can be designated as the index parameters. The regression equations presented within this report should then be used to estimate ratios of Clark unit hydrograph parameters relative to index parameters when simulating extreme events such as the Probable Maximum Flood or other design storms.

Index excess precipitation rates (the maximum excess precipitation rate realized during an historical event simulation) must also be estimated in order to apply the resultant variable Clark parameters. When observed data is used to calibrate the T_c and R parameters, the same historical event should be used to estimate the index excess precipitation rate. When observed data is not available to calibrate T_c and R parameters, a large historical event can be used to estimate the index excess precipitation rate.

Finally, prediction uncertainty should be quantified within all hypothetical event simulations. Due to the use of linear regression, uncertainty about any predicted T_c or R value is assumed to be normally distributed.

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