Effectively Utilizing Stochastic Hydrologic Loadings for Risk Analysis and Risk-Informed Design

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Abstract

The Bureau of Reclamation (Reclamation) currently utilizes stochastic methods in hydrologic modeling as part of a multiple-method approach to develop flood frequency estimates for use in risk analyses and risk-informed design for Reclamation dams and appurtenant structures. Broadly, the use of stochastic methods allows for improved characterization of uncertainty and development of more realistic hydrologic loadings over deterministic methods. Stochastic methods are employed within the modeling framework by sampling hydrological and meteorological datasets as well as model parameters using a Monte-Carlo simulation framework (MCSF). The MCSF allows for model inputs and parameters to be varied within physically realistic ranges for the study area and preserves co-variability of model inputs. The model can then be run thousands of times sampling from historical data and precipitation-frequency curves to simulate hypothetical annual maximum floods representing thousands of years of record. The underlying premise of this approach is that, the simulated hydrographs from a stochastic hydrologic model represent a broad frequency space and better characterize the inherent uncertainties in a flood frequency analysis. However, the large number of hydrographs developed as part of a stochastic study can pose technical, communication, and data management challenges to a team. While traditional methods typically presented one or a few hydrograph shapes, such as Probable Maximum Floods (PMFs), with which to support design or risk analysis, use of many realizations of potential flood hydrographs can make these processes far more complex. The methods and communication framework presented in this paper can support effective implementation of stochastic hydrologic modeling results in support of risk analysis and risk-informed design.

Introduction

Stochastic methods are being increasingly used by the Bureau of Reclamation’s Technical Service Center to improve characterization of uncertainty and to develop representative hydrologic loadings relative to deterministic methods. Simply put, stochastic methods treat model parameters and inputs as random variables rather than fixed values. Monte-Carlo simulation framework (MCSF) simulations samples from probability distributions of these parameters to better describe uncertainty in the model. Historically, deterministic methods were used to develop Inflow Design Floods (IDFs) including Probable Maximum Floods (PMFs) which were used to support design and risk assessment of Reclamation dams. Use of stochastic methods employed within the modeling framework by sampling hydrology and meteorology datasets and model parameters can allow for parameters and inputs to be varied within physically realistic ranges for the study area. This approach provides an improved representation of the potential variations in hydrologic conditions in a watershed of interest relative to those defined using deterministic methods where model inputs and parameters are largely specified as constants.

Flood frequency estimates from stochastic modeling studies are used to support risk analyses and inform design or operational modifications for Reclamation dams and appurtenant structures. The benefits associated with stochastic methods include an improved characterization of uncertainty in hydrologic modeling, improved confidence over estimates derived using limited datasets or deterministic methods, and potentially minimization of cost associated with physical design and/or operational modifications due to more refined hydrologic loads. The level of effort associated with stochastic models may not always be needed depending on the level of risk or confidence in estimates from an earlier study, so determination of the need for these methods should be defined in early project planning and risk screening studies. However, with these improved methods come challenges in application of the results of such stochastic models.

Relative to traditional methods in practice (e.g., use of IDFs or PMFs), stochastic hydrologic model results present communication and data management challenges and can limit their usefulness in situations where results are not effectively communicated. Traditionally, design and risk analysis team members were typically presented with just a handful of hydrographs for design and risk analysis. When presented with a suite of hundreds or thousands of hydrographs, team members are presented with the challenges of interpreting, understanding and using this information to support design and risk analysis. The response to these challenges may be for the team to either refer back to simplified methods; or, to improperly apply results to designs; or, to simply ignore the stochastic hydrologic modeling results.

To address these challenges, a robust communication plan between the team conducting the hydrologic modeling and those performing the design and risk assessment becomes a critical path - the critical path to bringing the stochastic hydrologic modeling results from theory to practice (Rubin et al. 2018). The objective of this paper is to define a robust communication plan for the development and application of stochastic hydrology models, and for them to be effectively implemented in risk analysis and risk-informed design decisions.

History of Hydrologic Loadings

Historically, dam design and analysis of risk was completed using a designated “design flood” such as an IDF or PMF. Use of this one design flood made for a simplified handoff of the design hydrograph from the hydrology team to the design or risk analysis team. This one flood was meant to represent extreme flood conditions with which the design team could be reasonably confident that the dam could handle the hydrologic risk posed to the facility. Often that one flood hydrograph shape was used to represent floods across the frequency spectrum by simply scaling it to estimates of frequency peak inflows or volumes. However, use of just one hydrograph for risk analysis is inadequate as it does not portray any of the uncertainty and natural variability of hydrologic responses, especially across the range of frequencies of interest for risk analysis.

Limitations with Use of “Design Flood”

There are several limitations and issues related to use of a single hydrograph that need to be addressed using more modern methods and data for a risk team to be confident in hydrologic loads being presented. The first issue is that, many design floods were developed several decades ago and therefore relied on minimal flow data available at that time. Often, stream gage sites were only established a couple of years before or even after a dam was built, limiting the observed floods available to support development of design floods. The second issue is the lack of quality precipitation datasets available at the time of design flood development. Over time, the spatial network of precipitation gages has grown as has the length of precipitation records. The availability of new precipitation data varied both spatially and temporally provides a higher confidence in precipitation-frequency estimates. Furthermore, many Probable Maximum Precipitation (PMP) and precipitation-frequency products available previously have been superseded by updated products leveraging new data. Many PMFs were developed based on limited precipitation data using Hydrometeorological Reports developed in the 1960s through 1990s. Potential increases in PMP estimates with recent data means that, many of the calculated PMFs may be underestimated (Gangrade et al. 2017). An overall limitation is that, hydrologic response is inherently complex due to the natural variability of factors such as soil conditions, storm intensity, storm aerial distribution, snowpack, etc. Natural meteorological variability further contributes to complexity of the response, including such variables as spatial distribution, temporal distribution, intensity, and temperature driving rain-snow partitioning. One single hydrograph realistically cannot capture the complexities of this natural variability of a hydrologic system.

Use of the historical “design flood” metric may generally seem overly conservative, which may be appealing from the perspective of risk reduction, but this poses potential problems from both risk and cost perspectives. From a risk perspective, previously estimated design floods may not actually be overly conservative. One such example would be if modern precipitation observations approach or exceed historically determined PMP values, such as in the case where extreme rainfall events approach or exceed existing PMP values (e.g. Kao et al. 2019). Another example would be if a different hydrograph shape representing the inherent variability of the hydrologic system may result in elevated risk compared to the design flood. From a cost perspective, use of an overly conservative flood load for design or modification can mean the difference between a project being cost prohibitive or feasible. These reasons highlight the importance of shifting from using deterministic design floods to using frequency floods developed from stochastic modeling approaches.

Hydrologic Hazard Curves

Hydrologic hazard curves are used to help determine hydrologic risk at a dam. The hydrologic load is the inflow hydrograph to the reservoir. These curves represent a relationship between hydrologic load and likelihood of that load or a response, otherwise known as annual exceedance probability (AEP). Common curve types (i.e. hydrologic loadings) include probabilistic peak inflow, maximum water surface elevation, volume, or duration of overtopping. Which curves are important is dependent on the characteristics of the facility, reservoir, watershed, meteorology as well as specific project needs and potential failure modes of concern. For the purposes of risk analysis within the Bureau of Reclamation’s dam safety program, the AEPs of interest are typically 1/10,000 to 1/1,000,000.

These curves are typically developed using multiple methods where data allows in order to maximize the credible extrapolation to support risk analysis. Use of multiple methods improves understanding of uncertainty in loadings estimates and allows for estimations for rarer AEPs. A summary of typical methods and limits of credible extrapolation are summarized in Table 1. The additional of stochastic hydrologic models can further extend the optimal credible extrapolation AEP beyond what is listed there. Typically, these curves are developed by fitting a distribution to an annual peak stream gage series and then extrapolating out to the AEPs of interest. As most stream gage records are ~100 years in length, extrapolating out to a 1,000,000-year event results in very high uncertainty. Paleoflood data usually improves confidence and reduces uncertainty of rare flood frequency estimates for hydrologic hazard curves, but uncertainty associated with paleoflood and non-exceedance estimates can still be high and may not be available at all sites. These methods are described in detail in Bulletin 17C (England et al. 2018). The use of stochastic hydrologic models in conjunction with stream gage records and paleoflood data can improve estimates for rare flood events and better represent the uncertainty to support risk analysis and any subsequent modification decisions.

Table 1. Hydrometeorological data types and extrapolation limits for flood frequency analysis (Adapted from Swain et al, 1998)

Type of Data Used for Hydrologic Hazard Curve Development	**Limit of Credible Extrapolation for AEP**	
	Typical	**Optimal**
At-site streamflow data	1/100	1/200
Regional streamflow data	1/750	1/1,000
At-site streamflow and at-site paleoflood data	¼,000	1/10,000
Regional precipitation data	½,000	1/10,000
Regional streamflow and regional paleoflood data	1/15,000	1/40,000
Combinations of regional data sets and extrapolation	1/40,000	1/100,000

Stochastic Modeling Overview and Challenges

Basics of Stochastic Hydrologic Modeling

The current practice to better understand uncertainty and represent the inherent variability of a watershed and its climatic forcings is through use of stochastic hydrologic modeling (Reclamation & USACE 2015; Sayers et al. 2014; England et al. 2014). Stochastic methods are employed within the modeling framework by sampling hydrological and meteorological datasets as well as model parameters using a MCSF. The MCSF allows for model inputs and parameters to be varied within physically realistic ranges for the study area and preserve co-variability of the model inputs. Examples of parameters that can be varied within the hydrology model include infiltration rates, subsurface storage, and flow timing. In addition to model parameters, meteorological forcings can be varied as well. Examples include rainfall duration and intensity, spatial distribution, and orographic parameters impacting snowfall and snowpack characteristics. Precipitation can be sampled from a regional precipitation-frequency curve along with its associated uncertainty and scaled to reflect representative spatial and temporal storm patterns for the study basin. Traditional design hydrology is typically far more simplistic and may not represent a suite of storms and associated responses. Variation of these parameters allow the hydrologic modeler to simulate a more realistic hydrologic responses compared to typical design hydrology methods.

Use of the MCSF allows for many thousands to hundreds of thousands of simulations to develop synthetic datasets representing thousands of years of hypothetical floods. Typical flood frequency curves are developed using a series of annual peak flows from a stream gage. However, in best-case scenarios, this is at most ~150 years of data (more often it is ~20-50 years of data). This can be further complicated by stream gages being discontinued or upstream regulation impacting peak timing and volume. Further, non-stationarity issues may impact gage records such as land-cover changes, fire disturbance, or climate change to name a few. The problem with extending a flood frequency curve out to a return period of interest for risk analyses from ~50 years of data is the large uncertainty associated with these estimates beyond credible extrapolation (refer to Table 1). This is where the PMF or IDF have historically played a significant role, but still with large uncertainty. Development of a large number of hydrographs allows for the team to better quantify uncertainty and gain insight into flood frequency characteristics beyond what is credible for analysis using only peak streamflow gages.

Another beneficial characteristic of stochastic modeling for risk analysis for dams is the ability to route the floods by varying initial water surface elevations in the reservoir. Historically, with use of a single hydrograph, reservoir routing analyses assumed a pre-determined set of initial water surface elevation, such as top of operating pool. In the Western US, reservoir water surface elevations can vary significantly and may not reach the top of active conservation pool value every year as many are purposed to store snowmelt (which varies annually). Therefore, use of a high initial water surface elevation that does not represent the seasonal variability inherent to reservoir operations may be unnecessarily conservative. Through the use of stochastic modeling, reservoir initial water surface elevation associated with the routing can be varied as well and tied to antecedent conditions such as snow pack, soil moisture, or simply randomly sampled from historical observations during the flood period of interest. This method of sampling the initial water surface from a historical distribution during flood season rather than simply choosing a design elevation allows for more realistic representation of the system and improved quantification of uncertainty.

The Challenges of Using Stochastic Modeling Results

One of the key challenges associated with the use of stochastic hydrology model results for risk assessment and design is knowledge and data transfer. Stochastic hydrology modeling often produces thousands to hundreds of thousands of hydrographs and reservoir routings. This can be a challenge for data transfer from the hydrology team to the rest of the risk analysis team or the design team. These datasets can be very large in size where old design hydrographs were typically table of hydrograph ordinates in report appendix and/or accompanying digital file. Datasets could be transferred via physical hard drives or network sharing; however, it presents the challenge of how to archive in a way that makes the data accessible for future use, especially if an archive management system has been focused on either paper copies or scans of documents. From a hydrologic loading perspective, these hydrographs could be used to simply develop frequency hazard curves, however, for more detailed studies, designers typically need to use hydrographs to simulate scenarios for modification of design or operations. This is where a data management challenge becomes a much larger communication challenge.

First, just the large amount of data relative to legacy methods like the PMF can be difficult to understand. Second, sometimes evolving past these legacy methods can be difficult, especially with those who have become accustomed to using a certain type of dataset (PMF, IDF, scaled hydrographs, etc.). Conversely, some design tasks do not readily lend themselves to stochastic approaches, necessitating development or selection of a small subset of hydrographs that effectively characterize uncertainties. This requires communication-oriented and opposite to the discussion above (designer to modeler) to clarify these needs. Communicating how stochastic hydrologic modeling works and why it is a more realistic representation of the system and better understanding of uncertainty becomes important. Finally, working with the entire multidisciplinary team to determine what are the needs at the facility of interest and how to use the hydrographs properly becomes one of the most important parts of the process.

Communication Framework

For stochastic modeling results to be properly implemented in a project, a robust communication framework should be developed at the onset of the project. This framework has been divided into three distinct phases: (1) planning, modeling, and baseline risk assessment; (2) design and analysis using results from phase 1 using a small subset of the generated hydrographs (e.g., 50-100, depending on project application); and (3) verification of acceptable risk from design/operations developed in phase 2 using the full suite of generated hydrographs (e.g., 10,000 or more).

Phase 1:

Phase 1 is the first important component of the study where the team is assembled, the stochastic modeling is completed, and the baseline risk is identified. Figure 1 illustrates the makeup of the teams involved in the process. A summary of the major tasks and subtasks associated with this phase are shown in the bulleted list below and in Figure 2. The overall plan for the project should be laid out during pre-project planning. The key players on the teams should be identified. The three elements of this type of study should include three teams – (i) hydrology team, (ii) risk analysis team, and (iii) design team. The hydrology team consists of members performing the stochastic hydrologic modeling, the design team uses the results to inform decision making in any design or operations modification, and the risk analysis team ultimately analyzes the overall risk at the facility using the results of the stochastic hydrology model. The teams need to communicate frequently throughout the process to meet the goals identified for the study. The hydrology study portion is complete when the hydrographs and hydrologic hazard curves are developed. The routing portion of the study (a task that could be shared between the hydrology and design team) uses all the hydrographs developed in the hydrology study and routes them through the reservoir. This includes sampling from a distribution of historic initial water surface elevations reflecting current operations and routing hydrographs through the dam using pre-determined existing routing scenarios. The result of the routing is generally a hydrologic hazard curve representing return periods for water surface elevations. Finally, after all the hydrologic hazard curves have been developed, the results go to baseline risk assessment to determine existing hydrologic risk.

- Pre-project planning
    - Identification of key team members

        - Identified in this report as “Hydrology Team,” “Risk Analysis Team,” and “Design Team”
        - Identify leads for each team for communication
    - Development of a communication plan
        - Who is leading what
        - How frequent should meetings be and what should the content be
        - What are the key deliverables
    - Identification of goals
        - What kind of study: design, modification, operations, or baseline risk analysis
        - What hydrologic hazard curves need to be developed (maximum water surface elevation, peak inflows, peak outflows, duration of overtopping)
        - What operations should be simulated
        - How will routing be completed? Inside stochastic model versus by team outside of hydrology team? This is dependent on the complexity of flood routing rules for a given facility as this may impact the computational requirements for data sets with significant numbers of hydrographs.
- Hydrology study
    - Development of inflow frequency hydrographs
    - Deliverable of hydrologic hazard curves for peak inflows and volumes
    - Archive of full set of hydrographs for future use
- Routing
    - Routing based on operations identified in pre-project planning
    - Review of initial reservoir elevations
- Baseline Risk Assessment
    - Use of hydrologic hazard curves to determine hydrologic risk.
    - Determination whether hydrologic risk at acceptable level
        - If yes, archive hydrographs for potential future use
        - If no, move onto phase 3
        - If design or operational modification project, move onto phase 2

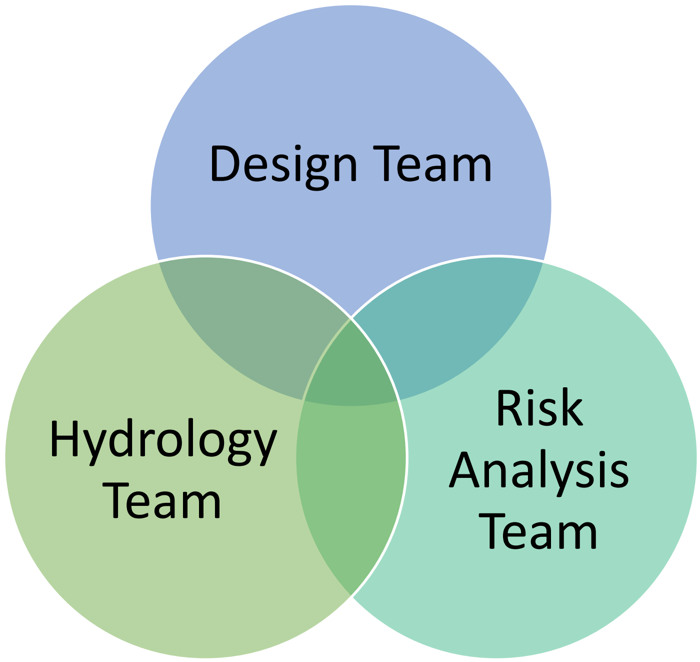

Figure 1. Venn diagram illustrating collaboration framework between the three teams involved in use of hydrologic loadings

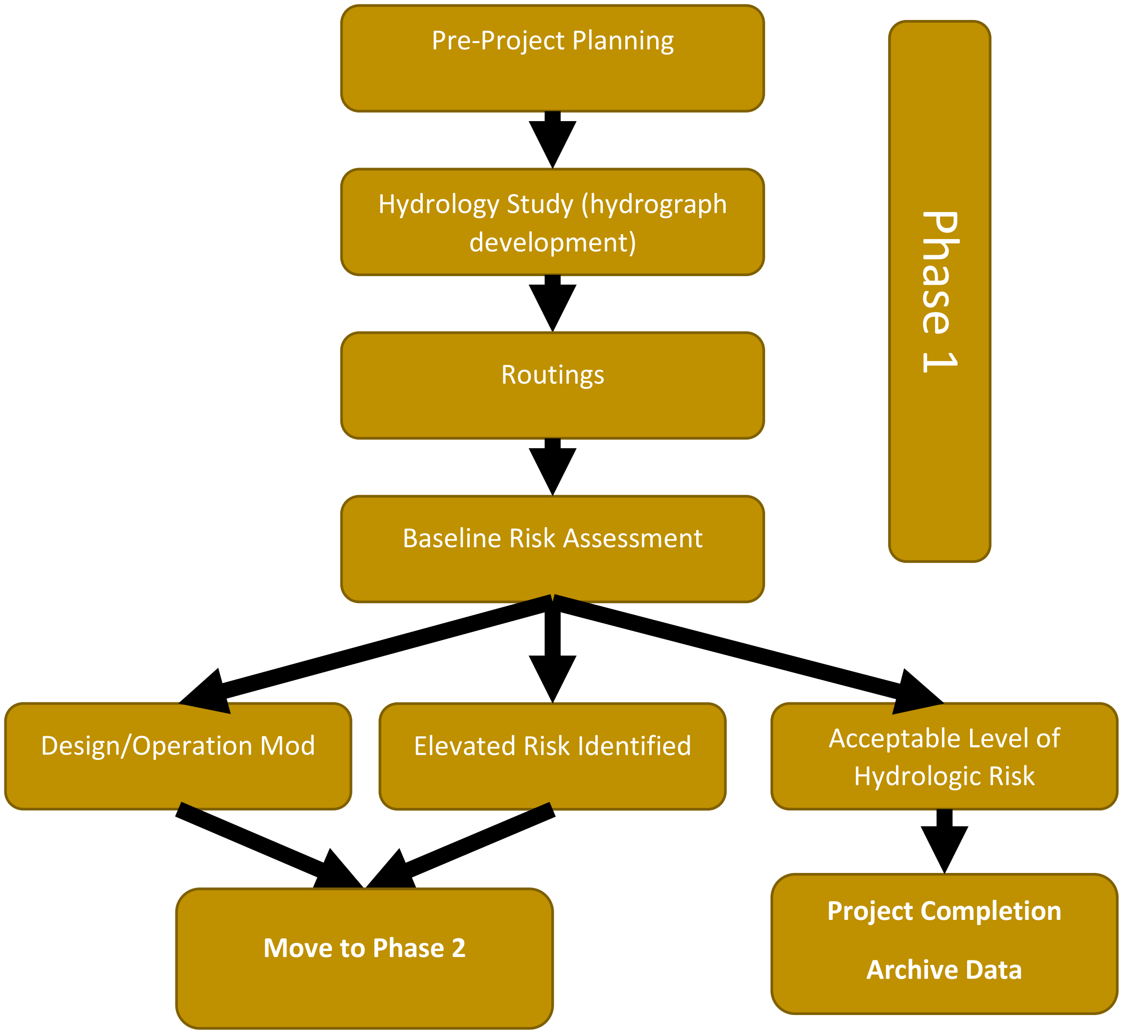

Figure 2. Summary of Phase 1 of stochastic hydrology study

Phase 2:

Phase 2 represents use of the hydrographs developed as part of Phase 1. Projects which either have identified elevated hydrologic risk for further analysis or have the need for any modifications move on to this phase. The main tasks and subtasks for Phase 2 are listed below and illustrated in Figure 3. The hydrology team in discussion with the design team extracts a subset of representative hydrographs around the AEPs of interest identified by the risk analysis and design teams for use in the design process, as described in the previous section. These hydrographs should be manageable for the design team to do screening level development of a variety of design and/or operations alternatives.

- Use of subset of hydrographs from Phase 1
    - Extract subset of hydrographs representative of important return periods identified by full project team – should be a small manageable number for use by design team
    - Extraction based on specific needs identified by design team

- Use subset of hydrographs to define design alternatives or operational changes
- Development of preliminary design/operations

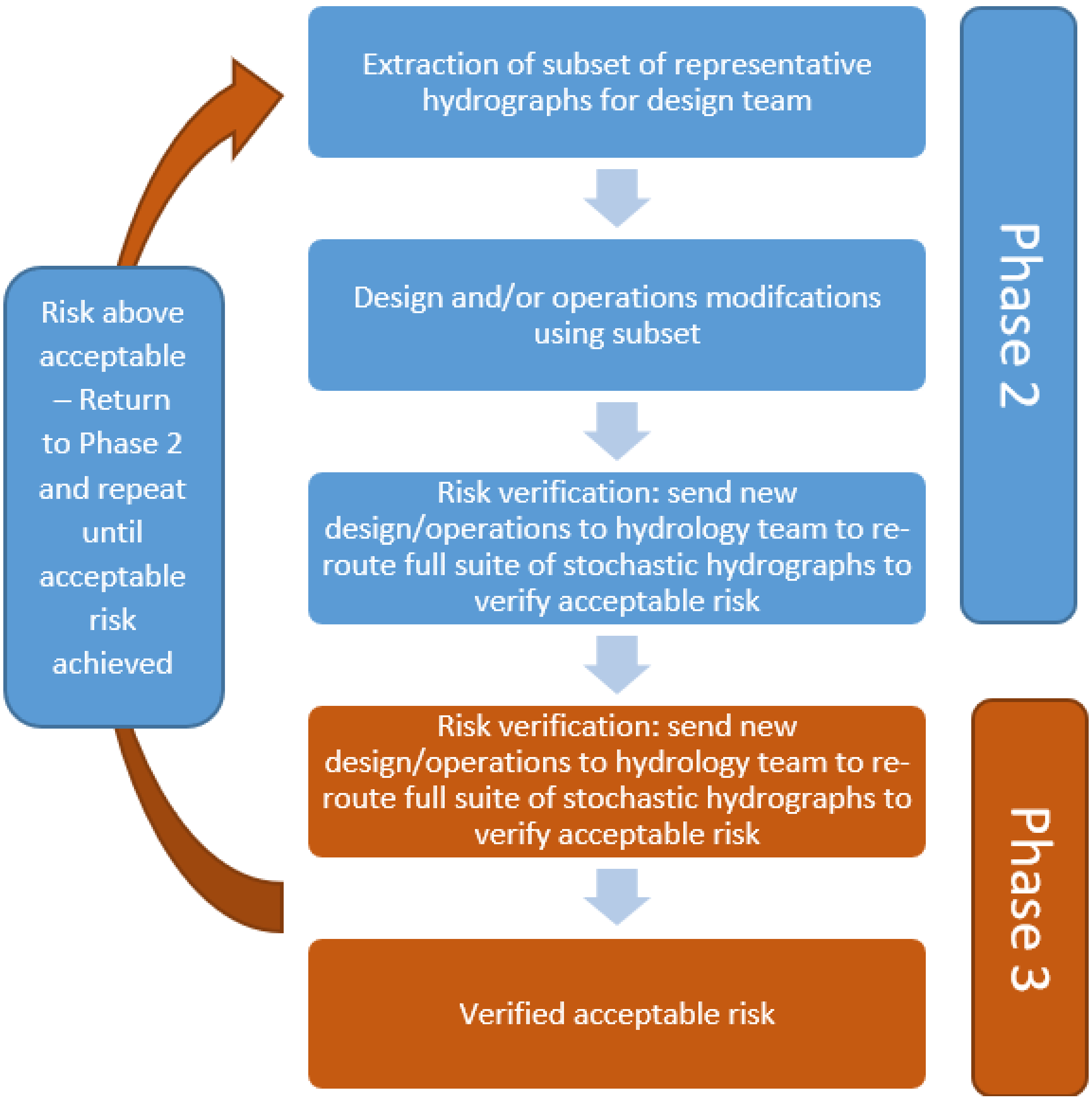

Figure 3. Summary of phases 2 and 3 of stochastic hydrology study

Phase 3:

Phase 3 immediately follows completion of design alternatives in Phase 2 (or as an iteration of the design process), illustrated in Figure 3. The goal of this phase is to evaluate if the risk associated with the design alternatives is acceptable for the risk analysis team. To complete the risk verification, the full suite of hydrographs developed in Phase 1 should be re-routed using the new alternatives to verify that risk is considered acceptable for the new design. If the full routings still indicate elevated risk, the hydrology and design teams return to Phase 2 to identify additional hydrographs or different criteria for hydrograph selection to support the design process.

- Verification of acceptable risk
  - Using new design alternatives or operations, hydrology team re-runs routings of full set of hydrographs from Phase 1 with routing settings (e.g., storage-discharge curves) provided by the design team
  - If risk level acceptable, design team moves forward
  - If risk level not acceptable, design team re-visits design or operational changes to re-submit to hydrology team for verification
    - Hydrology team may need to provide different or larger subset of hydrographs in support of the proposed design

Three hypothetical but representative project situations with variations in the communication framework are presented below:

- Through the screening level risk analysis, it was identified that Dam A had high hydrologic risk, although there was high uncertainty in the hydrologic loading estimates. A team is convened to perform a detailed stochastic hydrology study. The stochastic hydrology study changed the estimates and reduced the uncertainty in the hydrologic loads, bringing the hydrologic risk below guidelines. No modification of the facility is required due to hydrologic loads and the detailed study improved confidence and range of uncertainty in previous estimates.
- Dam B requires operations modification. Given high uncertainty in previous hydrologic hazard estimates, a detailed stochastic hydrology study is completed. A subset of hydrographs is extracted for the design team to develop operations alternatives. The operations alternatives are then used to route the entire suite of hydrographs developed during the stochastic hydrology study to verify acceptable risk. The risk analysis team determines that the operations modifications maintain an acceptable level of risk, allowing the design team to move forward with the proposed operations.
- Dam C is slated for a raise to increase storage capacity. Because of the high cost of the dam raise, a detailed stochastic hydrologic study is completed to maximize spillway capacity, other design consideration, and operations such that post-modification is risk-neutral. A subset of hydrographs is extracted for the design team to develop design alternatives. The proposed design alternatives are then used to route the full suite of hydrographs from the stochastic hydrology study for risk verification. The routing indicated that one of the alternatives results in elevated risk slightly above guidelines. The design team goes back to the hydrology team for additional hydrographs to support modification of the current design. Following modification, the design alternative is then re-verified through routing of the full suite of hydrographs, confirming acceptable risk of the modified design. The design team can then move forward with the proposed designs modifications.

Selection of Subset of Representative Hydrographs

While use of the full suite of hydrographs to support development of hazard curves for risk analysis is important, often the design team needs just a subset of representative hydrographs to develop design or operations changes. Choosing a subset of representative hydrographs can be a challenging task, especially if someone outside of the hydrologic modeling team is simply presented with many data files representing hydrographs with no guidance. Instead, the hydrologic modelers should work with the design team to determine the needs to support the design process. This process will look different for different facilities but following some general guidance can streamline this process.

The process to select a subset of hydrographs to support any design or operations modification can be divided into three parts. The first part is identifying a “risk window”, where the risk analysis and design teams identify what level of risk, or what return period, is critical for further analysis at that facility (Reclamation 2018). Also, what level of uncertainty is important to know? For example, accounting for factors such as population downstream and facility type, perhaps the risk analysis team is interested in looking at flood risk from the 100,000- to 1,000,000-year return periods. Perhaps the 90th percentile of uncertainty is deemed appropriate for risk analysis. This is then the risk window identified for further study.

The second part is up to the hydrology modeling team to determine what the inherent variability of the system is and how to identify hydrographs to represent the spectrum of natural variability for the watershed. For large and complex watersheds, such as those representing heavy snowfall mountainous regions adjacent to arid regions or those with extensive regulation, this may be a very large subset. For smaller, simpler basins, this will likely be a much smaller selection. For smaller projects, this could be completed simply with manual analysis and visual inspection. For larger projects, methods such as principal component analysis (PCA) could be used to identify clusters of similar hydrographs. Methods that could be applied beyond manual inspection or visual analysis include options such as PCA, self-organizing maps, or multivariate clustering approach (Hotelling 1933; Kohonen 1990). The applicable methods should be determined based on both an objective multivariate analysis coupled with judgement from the teams.

Finally, the design team should determine what is a manageable number of hydrographs to work with to refine designs. Ideally, this number should be large enough to be able to represent the natural variability of the system but small enough for the design team to efficiently test any modifications. If representative hydrographs are properly selected, proposed designs and operations should be able to pass the final verification process of routing the entire suite of hydrographs. Taking adequate care during this process can minimize the need to return to the hydrograph selection process after preliminary designs are completed.

Summary

When applied with multiple lines of evidence such as stream gage records and paleoflood data, use of stochastic methods in hydrologic modeling can improve confidence and improve characterization of uncertainty in hydrologic loading estimates to support risk analysis and the design process. Results from stochastic hydrology models are generally more realistic of existing hydrologic conditions in the watershed relative to deterministic methods employed in the past. Uncertainty is often better constrained for the extremely rare flood events of interest for management of risk. However, to foster use of stochastic modeling results in practice, additional effort must be given to planning and communication during the process. The large datasets and complex methods may appear confusing, overwhelming, or not conservative enough to those accustomed to traditional methods of developing hydrologic hazard estimates. The methods and framework provided here can be used in practice to support adoption of the methods as a standard practice in support of risk analysis and design for facilities.

The goal of this paper, through the communication plan and methods presented here, is to improve understanding of stochastic hydrologic model results and to provide a robust communication framework to utilize these tools in analysis, and risk-informed decision making. By implementing these tools at the onset of a project and throughout the study process, probabilistic flood hydrographs developed using stochastic hydrology models can be effectively implemented.

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