

# Rational Alternative to Linear Excess Shear Stress Formulation for Modeling Fluvial Erosion in Noncohesive Bank Materials Mobilized as Bedload

**David M. Waterman**, Assistant Professor, Department of Civil and Environmental Engineering, South Dakota School of Mines and Technology, Rapid City, SD, david.waterman@sdsmt.edu

**Kory M. Konsoer**, Assistant Professor, Department of Geography and Anthropology, Louisiana State University, Baton Rouge, LA, kkonsoer@lsu.edu

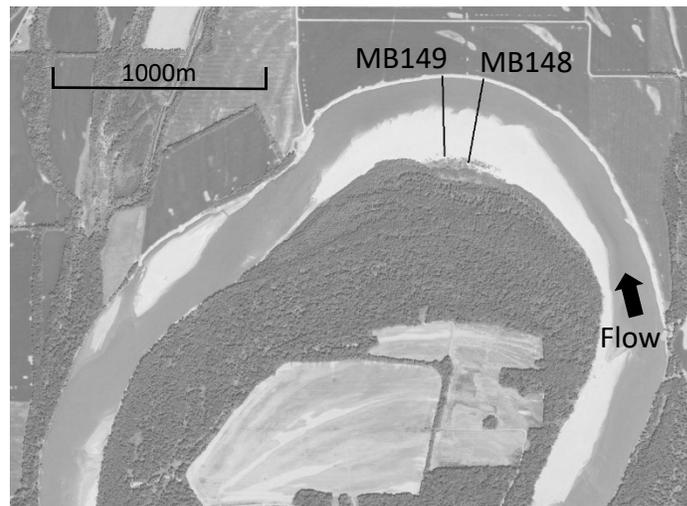
**Marcelo H. García**, Professor, Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, mhgarcia@illinois.edu

Realistic numerical modeling of stream morphodynamics requires realistic physical representation of bank erosion. The state-of-the-art bank erosion models that are primarily utilized in practice, BSTEM and CONCEPTS, are being incorporated into a wide range of morphodynamics models. These bank erosion models incorporate the key physical processes of fluvial erosion and mass failure. A knowledge gap has been identified in the research literature regarding the ability to properly parameterize the linear excess shear stress formulation used to quantify fluvial erosion when applied to coarse-grained bank material mobilized as bedload. Such bank materials are commonly encountered in composite river banks containing coarse basal material overlain by fine-grained soil.

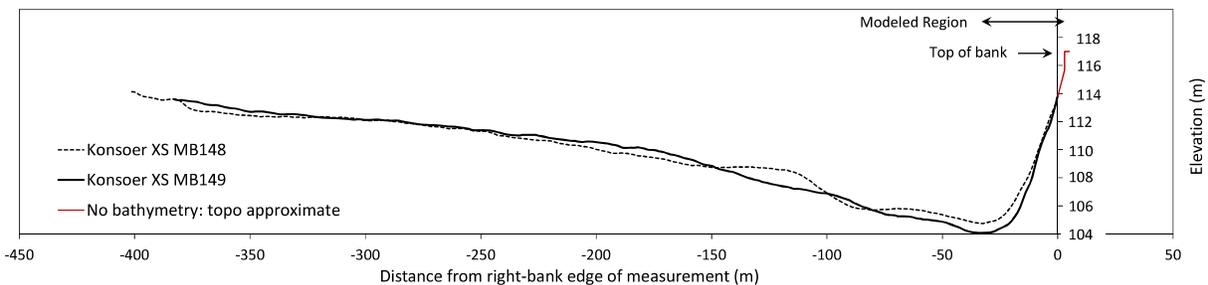
In this analysis, the linear excess shear stress formulation is abandoned in favor of the sediment mass conservation equation (Exner equation) with a constitutive relationship for bedload transport rate on a transverse slope. Only the divergence terms in the transverse direction are considered, which allows treatment of individual cross sections in the same modeling framework as BSTEM and CONCEPTS. Simple representations of the boundary shear stress distribution and the modification of the boundary shear stress vector due to secondary flow are implemented. The numerical treatment of the Exner equation is implemented using a finite difference method. If only the bank region is considered rather than the entire cross section, properly establishing the boundary condition at the base of the bank is of critical importance. A flux boundary condition yields realistic results when coupled with a simple bulk depositional model for the point bar region. The bulk depositional model allows the basal boundary to migrate a unit transverse distance when the time-integrated transverse flux of sediment past the thalweg equals the depositional volume required to cause the point bar to shift toward the thalweg by a unit transverse distance.

The current analysis is based on a field site on the lower Wabash River, located on the Illinois-Indiana border, at a bend known as Maier Bend. The river bank contains predominantly coarse sand bank material with only a thin upper layer of fine-grained soil. The data used was from spring 2011 bathymetry obtained during high flows and bank soil data obtained the following summer (Konsoer et al. 2016a; 2016b). A plan view of the site is shown on Fig. 1; cross sections labeled MB148 and MB149 are illustrated on Fig. 2. The cross-section geometry measured during high flow conditions in May 2011 for MB149 is used as the initial condition of the current model. To illustrate the characteristic behavior of bank deformation when comparing BSTEM to the current approach, the noncohesive layer is simplified with a single characteristic grain size equal to 1.0 mm coarse sand. This represents a spatially-averaged mean grain size in Meier bend sampled along the bank at an approximate elevation of 111 m, which corresponds to approximately half the bank height as measured from the high-flow thalweg (elev.  $\approx$  104 m) to

the top of bank (elev.  $\approx 117$  m). Flow conditions were set with water surface elevation at bankfull depth throughout the duration of the simulation. The BSTEM simulation was run by implementing the fluvial erosion module in 48-hour time intervals, after which the bank stability module was run to check for potential mass failure before proceeding with the next fluvial erosion interval. The critical shear stress ( $\tau_c$ ) for 1.0-mm sand was set equal to 0.71 Pa; the erodibility coefficient ( $k$ ) was set equal to 0.119  $\text{cm}^3/\text{Ns}$ . Both the  $\tau_c$  and  $k$  value were calculated by the BSTEM algorithms based on 1.0-mm sand.



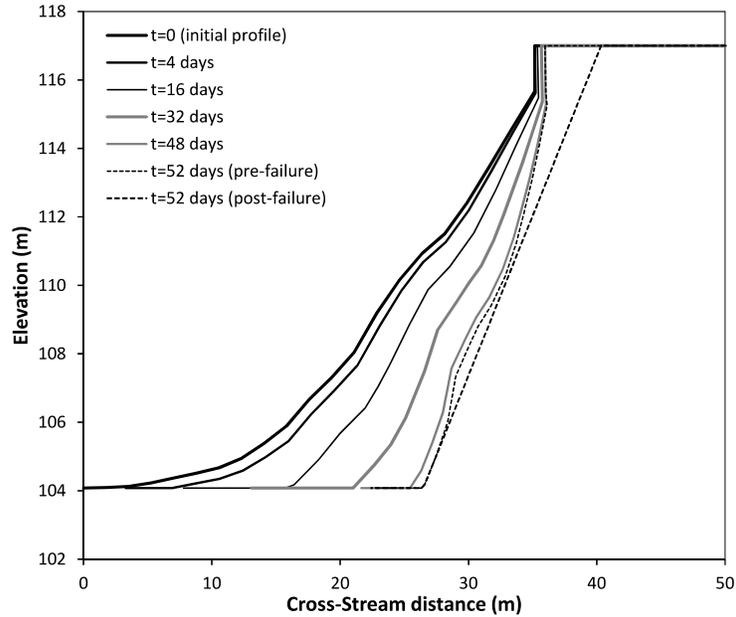
**Figure 1.** Plan view aerial photograph in fall 2011; source: Google Earth. Note that the field measurements of Konsoer et al. (2016a, 2016b) were from spring 2011.



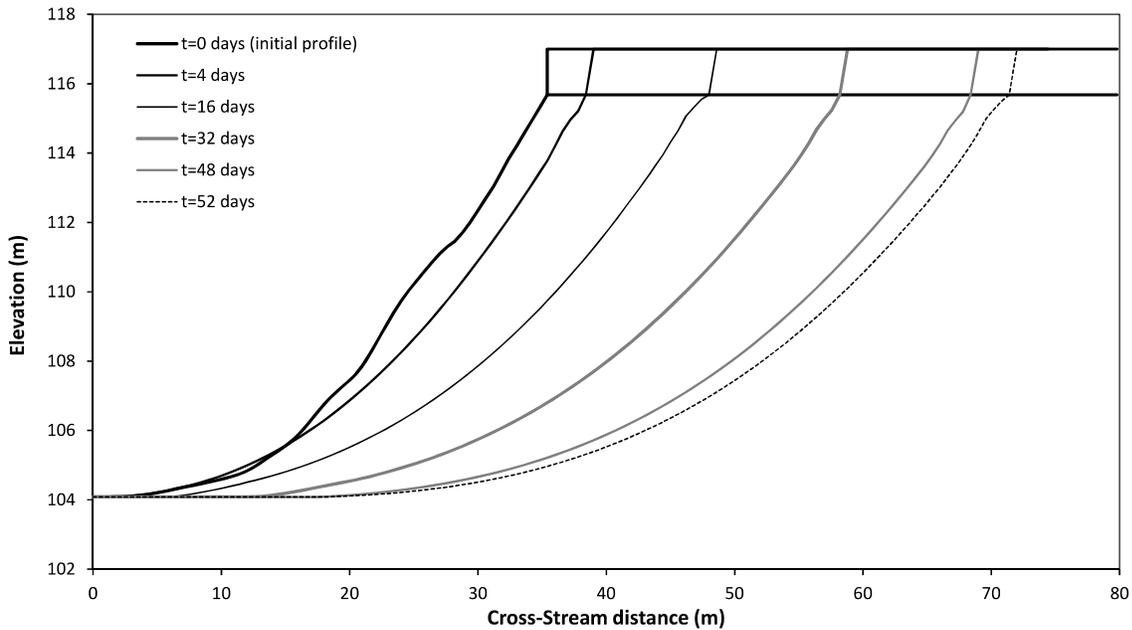
**Figure 2.** Cross-sections from bathymetric measurements near the bend apex; from the 2011 field measurements of Konsoer et al. (2016a, 2016b). Note that top of bank is at approximate elevation 117 m.

A side-by-side comparison of BSTEM results with the current model results are illustrated in Figures 3 and 4. Because the boundary shear stress increases with depth, the linear excess shear stress formulation for fluvial erosion yields continual steepening of the bank under steady hydrodynamic conditions near the bankfull depth until a mass failure involving nearly the full bank height results. On the other hand, the present model yields development of a concave upward bank shape in the noncohesive bank material that steepens with height above the thalweg; development of steep slopes exceeding the friction angle (imminent failure) only occur fairly high on the bank near the interface where the boundary shear stress drops below the critical value. Continuing evolution of the bank yields a translating bank profile with only modest deviations in profile shape due to shallow mass failures. The net migration of the toe of slope (thalweg) was similar between the two models: BSTEM produced a toe migration equal to 22.4 meters over the 52-day simulation period; the Exner-based model produced toe migration

equal to 18.0 meters over the 52-day simulation period. However, the migration distance for the remainder of the bank was grossly different between the two models. BSTEM yielded very modest top of bank migration (0.9 m) over the first 52-days, followed by a large-scale mass failure resulting in a net bank migration of 5.2 meters over the 52-day simulation period. The Exner-based model resulted in continuous migration of the top of bank, with a net bank migration of 36.6 meters over the 52-day simulation period.



**Figure 3.** Bank erosion results from BSTEM starting from the initial bank profile of XS MB149, with bank-full flow for 52 consecutive days (first mass failure).



**Figure 4.** Bank erosion results from the present modeling approach; same initial bank profile and same duration of bankfull flow as Fig 3.

The present model for fluvial erosion of coarse-grained materials is certainly more complex and computationally expensive than the linear excess shear stress formulation used by BSTEM and CONCEPTS; however, the model is more theoretically sound and requires no arbitrary parameterizations. When long simulation time scales are required, the finite-difference scheme can be replaced with a simple migration-rate formula based on an integration of the Exner equation over the bank region.

## References

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