Updating the Curve Number Method for Rainfall Runoff Estimation – Extended Abstract

Richard H. Hawkins, Ph.D., P.E., F. ASCE, F. EWRI, Professor Emeritus, University of Arizona. Tucson AZ 85721, rhawkins@ag.arizona.edu
Tim J. Ward, Ph.D., P.E., F. ASCE, F. EWRI, Dean and Professor, Manhattan College, Riverdale, NY, tim.ward@manhattan.edu
Donald E. Woodward, P.E., F. ASCE, National Hydraulic Engineer, USDA-Natural Resources Conservation Service, Retired, Gaithersburg, MD, dew7718@comcast.net

Introduction and Background

Origins

The well-known Curve Number (CN) method is used to estimate runoff depth, Q, from rainfall depth, P, and is used worldwide in a variety of applications. Since its genesis in the 1950s by the USDA Soil Conservation Service (now Natural Resources Conservation Service or NRCS), it has undergone numerous critical analyses on both practical and theoretical grounds. Originally intended to simply model runoff depth from design rain storms on small agricultural and rangeland watersheds, it has been opportunistically extended to a wide variety of conditions, including urban drainages, green roofs, solar farms, and continental scale river basin runoff.

In brief, the familiar CN method centers on the runoff equation

\[ Q = \frac{(P-0.2S)^2}{(P+0.8S)} \quad \text{for } P>0.2S, \quad Q=0 \text{ otherwise} \]  

where P is the event rainfall depth, Q is the median direct event runoff depth for the given P, and S is a measure of the hydrologic land condition, tied to of the maximum possible difference between P and Q. The CN is related to S by CN = 1000/(10+S) with S in inches. As shown in equation (1), 0.2S serves as an initial abstraction (Ia), or the amount of rainfall required before runoff begins. The S may vary from 0 to \( \infty \), thus CN inversely varies from 100 to 0. CN tables in handbooks give CNs for different soils (Hydrologic Soil Groups or HSGs) and land conditions.

The primary authoritative reference is the NRCS NEH-630 (USDA, 2003). In-house study reviews (Woodward, et al., 2003, 2004) identified issues in the CN method that should be addressed. A state-of-the-practice review was done under the auspices of ASCE in 2009 (Hawkins et al., 2009)

Curve Number Update Task Group

Within the past twenty years, there has been growing awareness of CN limitations and inconsistencies, and for the need to update the method. Accordingly, in late 2015, a joint ASCE-ASABE-NRCS Task Group, comprised of 16 volunteer members and co-chaired by the three authors here, was formed. Quarterly meetings over two years were held to consider needed revisions to review update status. In cooperation with NRCS, and with its support, Task Group delivered its report on October 1, 2017. Referred to as the Update here, it was based on experiences and advances in knowledge and data in watershed hydrology since the 1950s. It is still in agency review: to date (March 2019) changes in technical policy have yet not been endorsed by the NRCS nor incorporated in NEH 630 (USDA, 2003), the agency reference and
guide. The Update covers the four CN-related chapters (8, 9, 10, and 12) in NEH630, and is available on the ASCE-EWRI Collaborate site [https://collaboarate.asce.org/ewristatute] or from the authors. This presentation is intended to inform and solicit feedback from the audience and user community on the Updates and suggested revisions to the CN method.

Important assumptions and limitations in the Update were 1) uses of the CN method extend well beyond the original intended agency in-house applications; 2) data resources, computational abilities, and the quality of practitioners have greatly improved since the 1950s; 3) the work does not consider the many affiliated CN-using technologies of hydrograph generation, timing measures, daily time-step models, or geographic information systems; 4) it is centered on United States experiences; and 5) insofar as possible, strives for lumped model simplicity and consistency with prior offerings.

User experiences and data analyses from many sources since the 1950s has led to previously unappreciated findings and insights to rainfall-runoff processes in general. While originally a specific agency methodology, CN procedures are limited by being a subset of general hydrology, all the while complementing general hydrology. While not exhaustive, the major Update points are presented here.

**Findings and Recommendations**

**Runoff Response Behavior**

From the rainfall-runoff analysis of several hundred rainfall-runoff data sets, response patterns were found to fall into three distinct groups, not all of which are consistent with the CN runoff equation, i.e., equation (1). The simplest pattern found is a low linear reaction of the form \( Q = CP \), with \( C \) values (fraction of \( P \) that becomes \( Q \)) typically ranging from 0.005 to 0.05. Called the “Complacent” response, it is common even in some extreme rainfall conditions. This response is a version of the oft-used Rational Method.

Starting from low runoff complacent beginnings, there is, at some continuing higher rainfall threshold (\( P_c \), commonly 1.5-3 inches), a much higher incremental response fraction (about 0.6 to 1.0 in/in) that often sharply occurs. Such events, termed “Violent” response, are rarer, but can be quite consequential. Most data sets that have been examined – around 80% - show a “Standard” response, described further below, and which is compatible or compliant with CN procedures.

Following these observations of the different patterns, the Update recommends that the CN method not be applied to watersheds likely to exhibit Complacent or Violent runoff responses. An example is highly forested, base-flow watersheds, with high infiltration capacity that shows little evidence of overland flow. Karst topography with down-channel openings should also be excluded. In the Update, no alternative methods are endorsed for these conditions because they are not CN compliant rainfall runoff watersheds.

**Frequency Matching**

The original and predominant application of the CN method is the design calculation of event peak runoff from return period rainfall. With this in mind, the event rainfall \( P \) and runoff \( Q \) are matched by return period (rank-ordered matching). Preserving this in data analysis requires matching the rank-ordered \( P \) and rank-ordered \( Q \) in to unnatural pairs. Thus, most analyses use
ordered data, as opposed to the natural (Q resulting from P) data. This unusual ordered approach, called “frequency matching”, has been both useful and revealing.

**Asymptotic Behavior**

As evidenced by extensive analyses, data-based CNs – those defined by inverse solution of equation (1) for S and CN on ordered data - are rainfall dependent. When plotted against rainfall P, CNs are high for low rainfalls, but decrease with increasing storm size (depth P), and usually approach a steady state CN, referred to as \( CN_\infty \), which is asserted to be equivalent to the NEH 630 handbook reference values. This rainfall-runoff pattern is called “Standard” and is the most common in the situations encountered. The trend of CN to decline with P to a steady state value is called “Asymptotic” and as expressed by the following:

\[
CN(P) = CN_\infty + (100 - CN_\infty)\exp(-kP)
\]

where \( k \) (1/in) is a measure of the rate of change of CN with P. This is asymptotically consistent with CN method. This relationship is also generated through summed, distributed source-area-weighted area runoffs over an array of rainfall depths.

A subsequent simpler, and preferred, alternative formulation to equation (2a) is:

\[
CN(P) = CN_\infty + (100 - CN_\infty)\frac{P}{P_z}
\]

where \( \tau \) ranges from 0-1 and has a clear geometric representation on the CN:P plot. \( P_z \) is the threshold rainfall depth at \( Q=0 \), or \( \lambda S_\infty \), where \( \lambda = Ia/S \) (here 0.20 or 0.05). The two equations are equivalent with \( \tau = \exp(-kS_\infty) \), or \( k = -\ln(\tau)/P_z \).

**Initial Abstraction ratio**

As shown in equation (1), the CN method has used an abstraction ratio (\( \lambda \)) of 0.20. Studies over the past 20 years have shown the value of \( \lambda \) to be more appropriately about 0.05. To apply this in the Update requires revision of the runoff equation (1) to:

\[
Q = \frac{(P-0.05S_{05})^2}{(P+0.95S_{05})} \text{ for } P>0.05S_{05}, \ Q=0 \text{ otherwise}
\]

As inferred by the subscripting, the new \( S \) is **not the same as** the original \( S \) in equation (1), and the CN definition must be altered as well. A proposed transfer function in the Update between the long-used \( S \) value for \( \lambda = 0.20 \) and the value for \( \lambda = 0.05 \) is:

\[
S_{05} = 1.42S_{20}
\]

Subsequent discussions and analyses by Task Group members have now lead to an alternative formulation of (S values in inches):

\[
S_{05} = 1.3244(S_{20})^{0.089}
\]

This alternative formulation gives conversions very close to results from equation (4) (~1 CN unit) down to about a CN of 71. Equation (5) is now recommended, however.
Distributed-Weighted Runoff

In calculations, the Update recommends the use of distributed weighted runoff from individual CN source area fractions rather than the use an averaged, lumped CNs. That is,

\[ Q = \Sigma \alpha[(P-\lambda S_\alpha)^2/(P+(1-\lambda)S_\alpha) \text{ for all } \alpha \text{ and for } P>\lambda S_\alpha, Q=0 \text{ otherwise} \]  

(6)

where \( \alpha \) is the fraction of the drainage area represented by the given CN. This enhancement is within the capabilities of most rainfall-runoff modeling software. It should be noted that 1) it calculates runoff \( Q>0 \) for every \( P \) >minimum \( I_a \), but as a mean for that \( P \), and not a median; 2) it is more important with smaller storm events than with the higher extremes, and; 3) with back-calculation, it generates an asymptotic Standard response and thus is more in line with data-based findings. In short, it is a more realistic portrayal of expected watershed runoff with rainfall.

Secondary effects

In addition to the direct rainfall effect on runoff \( Q \), deviations, scatter, and variety in runoff due to other watershed and storm variables are expected but are not universally apparent. These include: 1) Land use effects are widely shown, but not for all storms, land uses, and sites. 2) Seasonal effects on CN have been found in some sites with accented moisture effects; 3) Event duration effects on CN are expected, but not widely demonstrated once the rainfall depth effects are excluded; 4) Event intensity distribution (i.e., storm pattern) seems to have inconsistent or minimal effects on CN runoff; 5) Prior rainfall effects, or watershed wetness is seen, but is not universal; and, 6) Effects of watershed slope on back-calculated CN is variable and is not consistent among the studies.

The prior-to-event rainfall criteria endorsed in early versions of the method to adjust for antecedent runoff conditions (ARC) were largely invalid and were discarded. The ARC concept itself was re-expressed as probability bands for ARCI (low runoff condition, 12%), ARCII (median runoff condition, 50%). and ARCIII (high runoff condition, 88%)

Table Curve Numbers

Successful application depends greatly on the choices of appropriate Curve Numbers for the contributing areas. Sensitivity studies show that the calculation (i.e., equation (1)) as more sensitive to variability in CN than to that of the rainfall \( P \). For the user, CN tables for different soils groups and land uses are provided in authoritative guides or by local approving jurisdictions. However, very few of the table entries are documented or based on analysis of field data. Thus, they might be realistically seen as conventions, or agreed-upon values offered in the absence of precise determination. They should be considered estimates based on best judgments of the tables’ authors.

Local Calibration

Because of the need for CNs for new/unlisted land uses or unusual watershed conditions, and for authoritative value, determination of CNs from local data sets is strongly encouraged. The original NRCS publications provided no clear instructions but did give an illustration of a median CN selected from annual flood events by graphical means. The Update provides a new procedure for estimating CN values from measured data, as outlined in the following steps. Given event rainfall and runoff values \((P, Q)\), with all \(0<Q\leq P\), use these steps in the procedure:
1. Rank order the P and Q separately (smallest to largest values)
2. Match the P:Q pairs based on the same rank-order.
3. Calculate the S (see equations (7) and (8)) and CN for each rank-ordered pair.
4. Plot CN against P with the presumption that the asymptotic standard shape results; if not, this not a CN compliant watershed or data set, i.e., use of the CN method is not recommended.
5. If asymptotic, find by either visual fitting or calculation the stable CN at high P. This is CN∞, taken to be the representative CN for the watershed, comparable to table values.
6. Select a representative CN and P in the P-sensitive drawdown part of the plot.
7. From this (CN, P) pair and CN∞, use equation (2a) or equation (2b) to determine the asymptotic coefficients k and/or τ.

The equations for S under the two Ia/S assumptions (0.20 or 0.05) are:

\[ S_{20} = 5(P+2Q-\sqrt{(4Q^2+5PQ)}) \]  \hspace{1cm} \text{(7)}

\[ S_{05} = 20(P+9.5Q-\sqrt{(90.25Q^2+20PQ)}) \]  \hspace{1cm} \text{(8)}

These two equations are dimensionally homogeneous so that P, Q, and S may be in either inches or mm, for example. However, in both cases, CN=1000/(10+S), with S inches. Optimum data-based values for k and CN∞ in asymptotic equation (2a) or (2b) can be determined by iterative methods and root-mean-squared-error (RMSE) criterion, using either CN or the resulting Q as the objective variable. This has been done extensively in the past for the Ia/S cases of both 0.20 and 0.05.

Local calibration of CN on local data is encouraged to deal with new land uses, such as green roofs, solar farms, or porous pavements. As demonstrated by recent wildfire events, Curve Numbers and time recovery parameters for freshly burned watersheds are badly needed. When produced, locally developed tables should be documented and made publically available.

Determining or verifying Curve Numbers by comparing experienced flood peaks or regional studies computed to modeled outputs poses a risk of confusing effects of hydrograph shape, dimensions, and timing measures with the underlying CN.

**Output Uncertainty**

Comparisons of data-derived CN findings to corresponding table values based on soils and land use usually show nonconformity. Consequently, runoff calculations based on table values are expected to contain uncertainty. Elementary estimates of CN uncertainty provided in the Update can be carried through to modeling calculations, thus providing a measure of scatter in the output. This step, seldom effected in current practice, is recommended, and should fit easily into existing modeling software.

**Practice, Research, and Development Suggestions**

**General**

Curve Number technology is still evolving and in need of improvements. Despite its obvious simplifications, development will contribute to an understanding of the general (i.e., non-CN) hydrology into which it conforms. Several issues are presented in the following sections.
Non-CN compliant watersheds

An important conclusion of the Update was that the CN method/equation does not correspond well to all rainfall-runoff watersheds. It should not be used in Complacent and Violent runoff settings. While the Update gives some suggestions, there is no practice-wide accepted set of alternative approaches. This shortcoming is an area that needs priority attention. An especially problematic case is a forested watershed. Under sufficient environmental conditions, tree-covered lands may display a distinct non-CN hydrologic response pattern (Complacent-Violent). Unfortunately, this is not a mutually exclusive relationship. Some non-forested lands show it as well, and some less-forested sites do not. Means of identifying these cases from land characteristics is needed.

Similarly, in the Violent case the threshold $P_t$ is needed, but not readily defined from site characteristics. The lack of Violent patterns in the data record will continue to hinder progress in better defining its causes. In general, and as suggested previously, there is a need for a priori definitions of the three runoff response types based on land and storm attributes.

Asymptotic Relations

The empirical nature of the Asymptotic-Standard procedure suggests unappreciated cause-and-effect connections between the land surfaces and the rainfall. Thus, this should be a productive focus area for further study and development. For examples: 1) What are the reasons/mechanisms for its very occurrence, and, 2) How might the asymptotic coefficients $k$ and $\tau$ (equations (2a) and (2b)) and be linked to the land characteristics and general hydrology?

History

The development and professional assimilation of the Curve Number method traces growing hydrologic knowledge and constantly changing user needs, and offers valuable lessons. While most of the original participants have passed, the technical-scientific-cultural-administrative evolution of the method should be documented as a noteworthy part of the history of hydrology. This should include the interfaces with its upland natural resources components.

Discussion

The different recommendations and findings from the Task Group and Update will be presented and discussed at SEDHYD 2019. The Task Group is still active.

Work Group Participants

The major authors and contributors have been, in alphabetical order: Hunter Birckhead, P.E., M.ASCE; James V. Bonta, Ph.D., P.E., F.ASCE; Donald Frevert, Ph.D., P.E., D.WRE (Ret), F.ASCE; Claudia Hoeft, P.E., F.ASCE (USDA NRCS liaison); Richard H. Hawkins, Ph.D., P.E., F.EWRI, F.ASCE (Task Group chair); Rosanna La Plante, P.E., M.ASCE; Michael E. Meadows, Ph.D., P.E., F.ASCE; Julianne Miller, A.M.ASCE; Steven C. McCutcheon, Ph.D., P.E., D.WRE (Ret), F.EWRI, F.ASCE; Glenn Moglen, Ph.D., P.E., F.EWRI, F.ASCE; David Powers, P.E., D.WRE, F.ASCE; John Ramirez-Avila, Ph.D., ING, M.ASCE; E. William Tollner, Ph.D., P.E., M.ASCE, F.ASABE (American Society of Agricultural and Biological Engineers [ASABE] representative); Joseph A. Van Mullem, P.E., M.ASCE; Tim J. Ward, Ph.D., P.E., F.ASCE,
F.EWRI (Task Group co-chair), and Donald E. Woodward , P.E, F.EWRI, ASCE (Task Group co-chair).

The Update was reviewed by a select, three-member external review team composed of Wilbert Thomas, Jr. (Michael Baker International, and USGS retired), Dr. Bill Elliot (U.S.D.A. Forest Service), and Karen Kabbes (Kabbes Engineering). The authors Task Group thank them for their input.

References


