

# Beyond Bankfull

**Peter Wilcock**, Professor, Department of Watershed Sciences,  
Quinney College of Natural Resources, Utah State University, Logan Utah  
wilcock@usu.edu

## Abstract

River channels must deal with the full range of water and sediment supplied to them. So should those who work to assess or design river channels. A rigorous channel design approach requires specification of key drivers, articulation of desired channel behavior, and development of a suitable design that links the two. In terms of sediment processes, the primary drivers are water and sediment supply. Channel behavior is defined by the sediment balance and mobility of the stream bed. Rather than a rational forward process based specified channel behavior, channel design has generally invoked broad empirical correlations to set channel dimension and to define the flow that fills the channel – the bankfull flow. Such an approach implies that selection of the correct bankfull discharge and its correlated channel dimension will produce desirable channel conditions. This approach is inevitably a black box ... if the channel is sized to the correct flow, then good things will happen.

Tools are available for a direct design approach that links water and sediment supply to desired channel behavior. Some decades ago, an approach promulgated by R. Copeland of the USACE used a specified water and sediment supply to determine the slope necessary for sediment balance (Copeland et al., 2001). The slope is calculated for a range of channel widths, relegating the use of broad empirical correlations (e.g. channel width as a function of drainage area) to its proper role: one might advisedly select a channel width from such data, but the slope (and depth) for that channel have already been determined such that the channel will transport the sediment supplied with a specified flow. This step forward left one key matter unresolved: selection of the design, or bankfull flow for which the calculations are made. The search for a ‘correct’ bankfull flow has consumed fluvial geomorphology and river engineering for many decades (e.g. Wolman and Miller, 1960; Copeland et al., 2000), despite evidence that a single correct discharge does not, in fact, exist. The need for careful consideration of the linkage between the water and sediment drivers and channel behavior is likely to be particularly important for streams that are far from transport equilibrium, which may well be those requiring more immediate attention. The key step added in this paper is to point out that a single bankfull flow is not a requirement for completing a channel design. Rather, the full range of flows (and their associated sediment supply) can be used to estimate the graded channel slope that will transport the sediment supplied over the full range of flows. We *can* move beyond bankfull. Combined with companion relations for a static (or threshold) channel, a channel design approach can be defined linking water and sediment supply to desired channel behavior.

This paper outlines that design approach based on three essential questions: (i) should the channel store or evacuate sediment, (ii) should the bed of the channel be mobile at a specified discharge, and (iii) a combination of (i) and (ii), which we term overcapacity threshold. Inasmuch as there is considerable uncertainty in specifying future water and sediment supply, as well as in the calculation of sediment transport rates, the paper also discusses strategy for accommodating that uncertainty.

## Introduction

It's time to move on. For too long, channel assessment and design have been based on some notion of a "correct" discharge that just fills the channel. This discharge might be determined from a variety of metrics, including flood frequency, drainage area, and various field indicators. That this approach is flawed is immediately evident from changing fashion: the "correct" flow magnitude over the years has decreased gradually, then suddenly, from order two-year flood to flows that occur every year. The fundamental problem lies in the absence of a linkage between channel dimension and desired attributes of channel behavior. One merely argues that selection of the "correct" discharge will cause good things to happen, whether the good things are based on channel dynamics, riparian habitat, aquatic life, or appearance. This is backwards. Correct design begins with drivers and objectives and then explores explicit linkages between the two to evaluate the options for meeting the objectives. For flow and transport in a river channel, the starting point is to ask: "what is the supply of water and sediment and what do you wish to do with them?" (Wilcock, 1997). There are two basic elements to sediment behavior in river channels: the mobility of the bed material (zero mobility = threshold channel) and the balance between sediment supply and transport capacity (the Lane Balance provides a conceptual, but not predictive model of sediment surplus or deficit). Channels fall along the full spectrum of static to mobile beds and channels may accumulate or evacuate sediment. These are the essential "what do you wish to do with them" attributes which then support other stream objectives such as ecosystem change, habitat type and amount, infrastructure and flood protection, recreational use, and appearance. An exactly balanced sediment budget is the graded slope from the fluvial geomorphological canon. It is worth emphasizing that a successful design need not have a static or a mobile bed, nor must the channel be at grade. These elements of channel behavior are determined in order to meet the broader objectives of the design.

The solution to the threshold and mobile-bed problems can be evaluated on a single chart with two curves – one threshold, one mobile-bed – giving channel slope as a function of channel width. Tools to develop such a chart have been widely promulgated (for example by NRCS and USACE). By first determining the slope (and depth) required for desired channel behavior, the selection of channel dimension (width) is moved from the beginning of the analysis to its proper position at the end. One would be well advised to consult broad correlations between channel width and various independent metrics such as the flood record or drainage area. But for any width selected, the channel slope and depth have been determined as those needed to provide the desired channel behavior given the water and sediment supply.

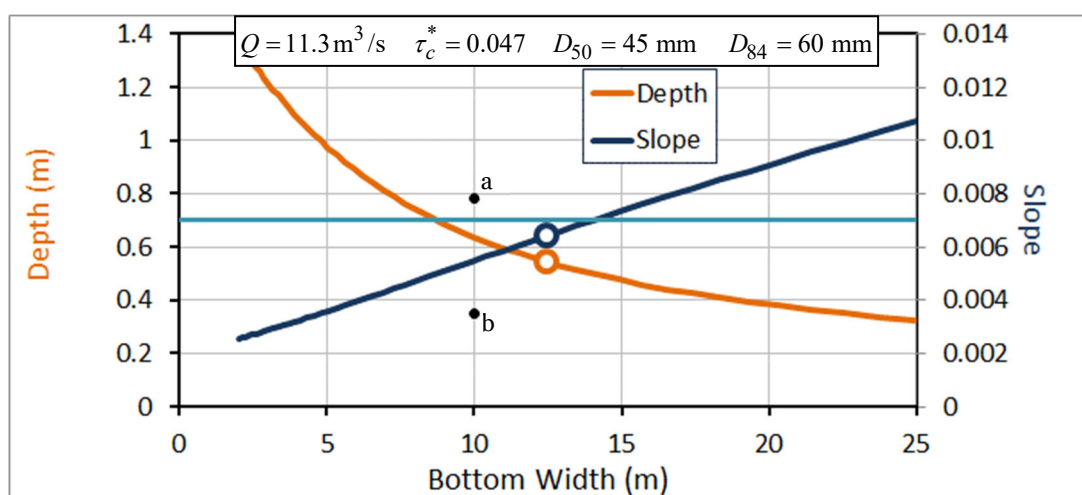
Which brings us to the last piece of the puzzle: What discharge to use? If one must specify a single "correct" discharge (and associated sediment supply) for the design charts, one is left not knowing whether more frequent smaller flows or rarer larger flows might, in fact, produce undesirable channel behavior. Selecting a single "bankfull" or "dominant discharge" is simply not necessary. It is possible to determine the graded slope (or the amount of sediment stored or evacuated) for the full range of discharges that a channel will experience. The river must deal with the full range of discharge – so should the channel designer.

Once the threshold and graded slopes are determined, channel assessment and design can proceed to consider the desired channel behavior as well as the risk associated with uncertainty in the inputs and calculations. Such an approach might appear to be fraught with uncertainty and freighted with effort. Wouldn't it be simpler to pick a currently popular "correct" discharge and go from there? The answer depends on one's faith – a forecast without prediction – that the "correct" discharge will produce the desired result. Maybe that will work. Maybe not. Better to

specify the drivers and predict the channel behavior, even with uncertainty. The difference is that consideration of transport and channel behavior is explicit in the latter case, rather than hoping for the best.

## Previous Work

The essentials of channel behavior can be defined in terms of the mobility of the bed material and the balance between sediment supply and transport capacity. A threshold channel is designed such that the material in its bed and banks is immobile at a specified flow. This type of channel behavior has been successfully analyzed for well over a century and has been most widely promulgated by the USDA NRCS (formerly SCS). A straightforward presentation of the method is given in Chapter 8 of the NRCS Stream Restoration Handbook (NRCS, 2007; Figure 1). The example problem in that chapter specifies a 25-yr flood as the design discharge, meaning the bed of the channel should be on the brink of motion at that flow. The problem calls for a trapezoidal channel with defined side-slope, specified bed  $D_{84}$  and  $D_{50}$ , and indicates that the valley slope is 0.007.  $D_{84}$  is used in the Limerinos roughness model and  $D_{50}$  is used with a critical Shields Number  $\tau_c^* = 0.047$  to find the bed shear stress. The problem selects a bottom width of 12.5 m. Figure 1 shows the solution not only for that width but for a wide range of channel width for consistency with the mobile-bed solutions shown below.



**Figure 1.** Threshold channel solution for case presented in NRCS NEH654 (p. 8-40). Published solution shown in circles for trapezoidal channel with bottom width 12.5 m and side slope  $m = 3$ .

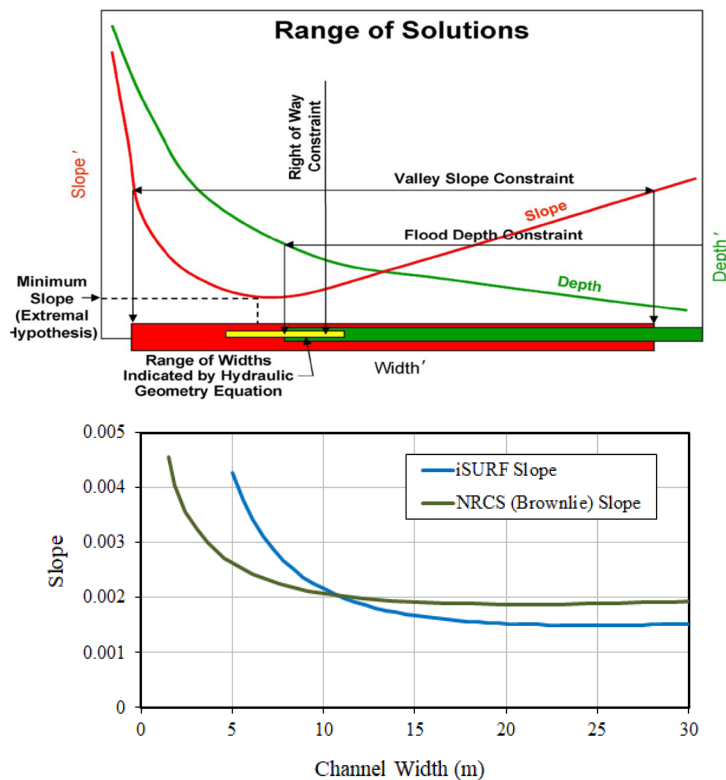
Points a and b on Figure 1 illustrate the nature of risk in the design of a threshold channel. For a bottom width of 10 m and the parameter values as defined, the threshold slope is 0.055. If the true bed  $D_{50}$  were 64 mm, rather than 45 mm, the threshold slope would be 0.077 (Point a). The calculated slope of 0.055 is conservative. That is, it is smaller than that needed to produce incipient motion at the design discharge and the bed is static at the design flow. The channel at a slope of 0.055 will also be longer than that needed. If, on the other hand, the true  $D_{50}$  were 32 mm, the true threshold slope would be 0.035 (Point b) and the calculated slope would be too large (a similar result arises if one uses  $\tau_c^* = 0.03$ , rather than 0.047). In that case, the calculated slope of 0.055 not conservative; sediment mobilizes at the design flow, failing to meet threshold conditions. For a threshold channel, risk is represented by a slope larger than needed for incipient motion at the design flow.

Copeland *et al.* (2001) provide a solution for mobile channel design in which transport capacity is matched to sediment supply (Figure 2, top). For a specified design discharge and its associated sediment supply, the approach uses relations for channel hydraulics and sediment transport rate to find the channel slope (red line) and depth (green line) necessary to transport the supplied sediment with the available flow. By solving for a wide range of channel width, the approach places the specification of channel width (and other channel dimensions that scale with width) at the end of the analysis, rather than at the beginning as might be the case in template-based approaches. With the problem solved for sediment balance, one can consult hydraulic geometry or other local relations for channel width (yellow range on abscissa of Figure 2), but the slope (and depth) associated with the choice of width is already determined from the specified water and sediment supply.

This mobile-bed approach is illustrated as Example 2 in Chapter 9 of NRCS NEH654 (p. 9-39; Figure 2; bottom). Sediment supply is determined from transport calculations at a specified discharge in a supply channel with specified slope, geometry, and bed material. The solution presented in the NRCS example uses the Brownlie (1981) relations for roughness and total sediment transport rate. This approach can be taken a step further using a many-fraction, mixed-size transport model. Wilcock and DeTemple (2005) showed that the Wilcock-Crowe (2003) transport model can be solved in the inverse, wherein the bed shear stress and bed surface grain size are determined based on a specified sediment transport rate and grain size. For a specified channel geometry and water discharge (thus water and sediment supply are fully specified), basic hydraulic relations can be used to find channel slope and depth, similar to the Copeland approach.

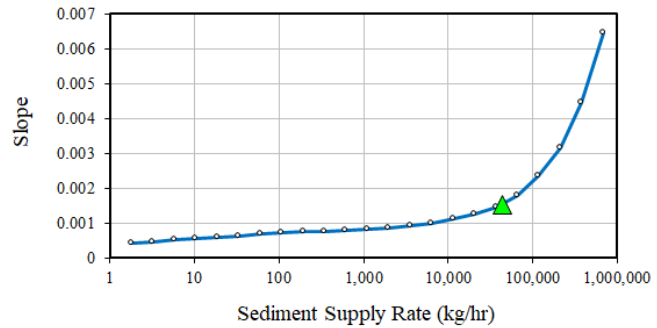
The approach, termed *iSurf*, is also shown in Figure 2 (bottom). The difference between the Brownlie and *iSURF* solutions in Figure 2 (as well as points a and b in Figure 1) illustrate that uncertainty arises not only from input uncertainty, but from the choice of transport and hydraulic formulas.

A useful extension of the inverse approach is to solve for the slope needed to transport a wide range of sediment supply rates for a specified discharge. Figure 3 presents the same *iSurf* solution as given in Figure 2 (green triangle) as part of a trend showing the slope required to transport the same sediment supply grain size over a wide range of supply rates. The curve in Figure 3 is essentially a total transport curve for a specified transport grain size and water



**Figure 2.** Channel Design Charts Top: Schematic of Copeland *et al.* 2001. Bottom: Solution of Example 2, NRCS NEH654 (p. 9-39) using Brownlie (1981) and *iSURF* (Wilcock and DeTemple, 2005) solutions.

discharge. Slope stands in for shear stress and one may recognize the characteristic form of a sediment transport relation (in this case Wilcock-Crowe) with a critical or reference stress (or slope) for incipient motion. The utility of Figure 3 is that it demonstrates that the slope needed to transport a specified sediment supply at a specified discharge is rather insensitive to the sediment supply rate until one reaches relatively large transport rates, such as that specified for the NEH654 example illustrated in Figure 2.



**Figure 3.** Variation of the slope needed to transport the sediment supply grain size at the specified discharge used in Figure 2, for a wide range of sediment supply rate.

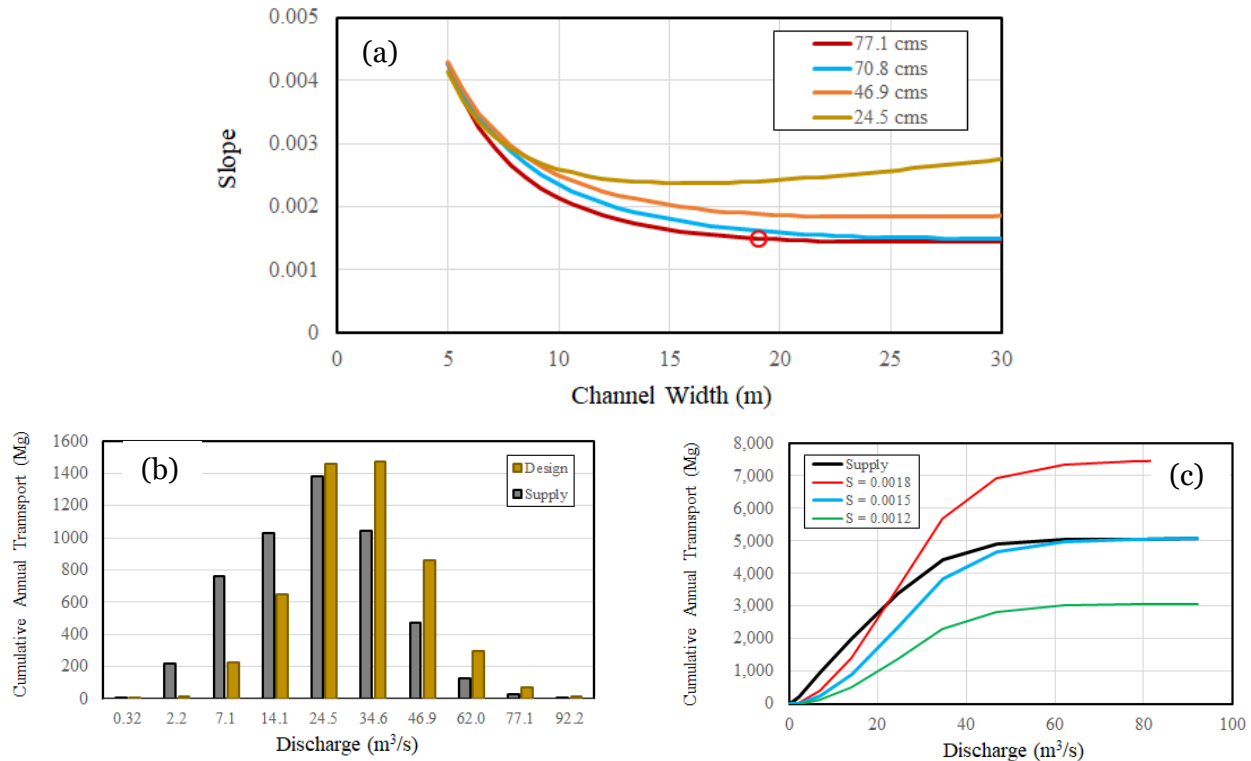
## The Missing Piece: Design Discharge

All of the approaches discussed thus far require specification of a single water discharge that fills the channel. In the case of threshold channels, design risk is typically defined in terms of flood frequency, such that selection of a design discharge can be based on annual risk. For example, if a 25-yr flood is chosen as the design discharge as in Figure 1, a channel built to the calculated width, depth, and slope is designed to have a 4% chance of failure each year. For an alluvial channel, the choice of design discharge has no such strategic basis and has been the subject of considerable investigation over many decades. Concepts such as dominant discharge and effective discharge have been introduced and “bankfull” discharge has come to collectively represent the single discharge that is used to determine desired channel conditions. A discharge of a given flood frequency is commonly used to specify the bankfull design channel.

But why? The channel must experience the full range of flow and associated sediment supply. It strains credibility to suggest that a particular flood frequency – or any other simple choice – would produce the desired channel behavior for the full range of water and sediment supply that rivers experience. Further, channels deemed to need repair are likely to be in transport disequilibrium, suggesting that careful attention to the supply of water and sediment and its transport are merited.

The issue is illustrated in Figure 4. The example expands on the example of NRCS NEH654 (p. 9-39) shown in Figure 2. A full flow duration curve is defined for the supply channel and the sediment supply rate and grain size for each flow is calculated. The width/slope solutions for a graded channel at four different discharges are shown. Slope clearly varies not only with channel width but with the choice of discharge. If the channel were designed with a slope of 0.0018 and a channel width of 20 m, the transport capacity at 70.8 m<sup>3</sup>/s would exceed that needed to transport the sediment supply at that flow, producing sediment evacuation. But the transport capacity at 46.9 m<sup>3</sup>/s (which happens to be the effective discharge for the specified flow duration curve and calculated sediment supply) would be smaller than needed to transport the sediment supply, leading to sediment accumulation. A larger channel slope increases the fraction of the total load that can be transported over all flows. A smaller slope decreases the fraction of the total load that can be transported over all flows. The slope that just balances sediment transport capacity to sediment supply, *over the full range of discharges*, is the graded slope of Mackin (1948) and many others. This graded slope can be determined, even for mixed-size sediment transport in which there is complex internal adjustment between the grain size of the transport and of the bed at different transport rates. For a channel width of 19 m, the graded

channel slope is slightly greater than 0.0015 (Figure 4a). Figure 4b shows that the sediment supply exceeds the transport capacity of the design reach at smaller flows and the transport capacity exceeds the supply at higher flows, to produce an annual balance. Stroth *et al.* (2017) demonstrated a similar method that can provide the graded slope that will transport the supplied sediment over the range of flows.



**Figure 4.** (a) *iSurf* solution for the slope needed to transport the supplied sediment at four different discharges for the expanded Example 2 of NRCS Ch 9. (b) Annual sediment supply and transport in the design reach for a width of 19 m and a slope of 0.0015, which balances supply and transport capacity over the year. (c) Sediment supply and annual transport for three different slopes, cumulated over discharge

Note that a graded slope is not a design requirement (although many channel design approaches use a graded condition as an implied requirement). For example, in a setting with a substantial sediment supply, gradual accumulation of sediment in the valley bottom may, in fact, be desirable in that it could drive channel shifting, producing a dynamic, changing habitat mosaic that could be preferred for stream ecosystem restoration. For the example shown in Figure 4, a design slope of 0.0012, smaller than the graded slope, results in accumulation of about 2,000 Mg over the year. A mass of 2,000 Mg roughly corresponds to the mass of a large point bar in a river of width 19 m, which may be a desirable feature of a channel design. A design slope of 0.0018 results in potential evacuation of 2,300 Mg over the year. Consideration of designing channels to intentionally accumulate or evacuate sediment will be taken up further below, when we consider uncertainty and risk in channel design.

## Threshold and Mobile Bed Combined

The utility of showing both threshold and mobile-bed solutions as a function of channel width is that both can be combined on a single figure. Figure 5 presents an illustrative case. For simplicity, consider a trapezoidal channel with the indicated bed material and sediment supply. For a discharge of  $100 \text{ m}^3/\text{s}$ , a threshold and mobile-bed solution can be found. To illustrate the effect of uncertainty, channel roughness, critical Shields Number, and bed material are varied by  $\pm 10\%$  and a second sediment supply case with increased discharge ( $160 \text{ m}^3/\text{s}$ ) and sediment supply ( $60 \text{ t/hr}$ ) are shown. The threshold and mobile-bed solution are shown together in the lower right panel of Figure 5, indicating that for most of the range of channel width, the threshold slope is larger than the mobile-bed slope. Even accounting for uncertainty in the specified parameters, there is a range, labelled *Over-capacity threshold*, for which the slope is smaller than required to mobilize the bed yet larger than required to transport the supplied sediment. In this range, although the sediment balance is in deficit, the flow is unable to entrain sediment from the bed, such that sediment evacuation and incision does not occur.

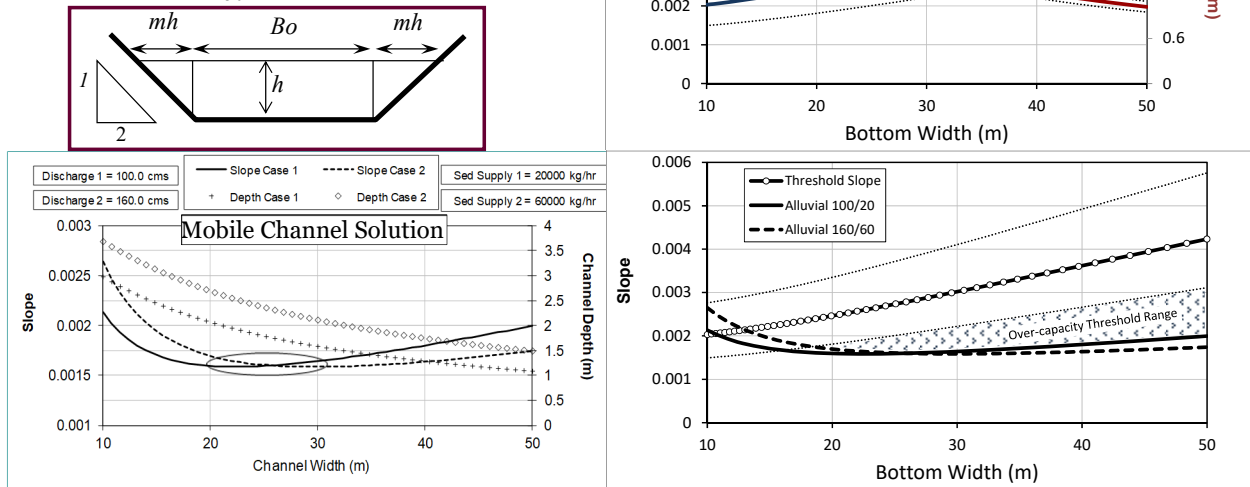
$Q = 100 \text{ m}^3/\text{s}$ ; Roughness:  $n = 0.04$  ( $\pm 10\%$ )

Critical stress:  $\tau_c^* = 0.03$  ( $\pm 10\%$ )

Bed Material:  $D_{50} = 64 \text{ mm}$ ,  $D_{84} = 128 \text{ mm}$  ( $\pm 10\%$ )

Sediment Supply:  $Q_s = 20 \text{ t/hr}$  { $60 \text{ t/hr}$ }

Supply Grain Size:  $D_{50} = 13 \text{ mm}$  ( $0.5 \text{ mm} - 128 \text{ mm}$ )



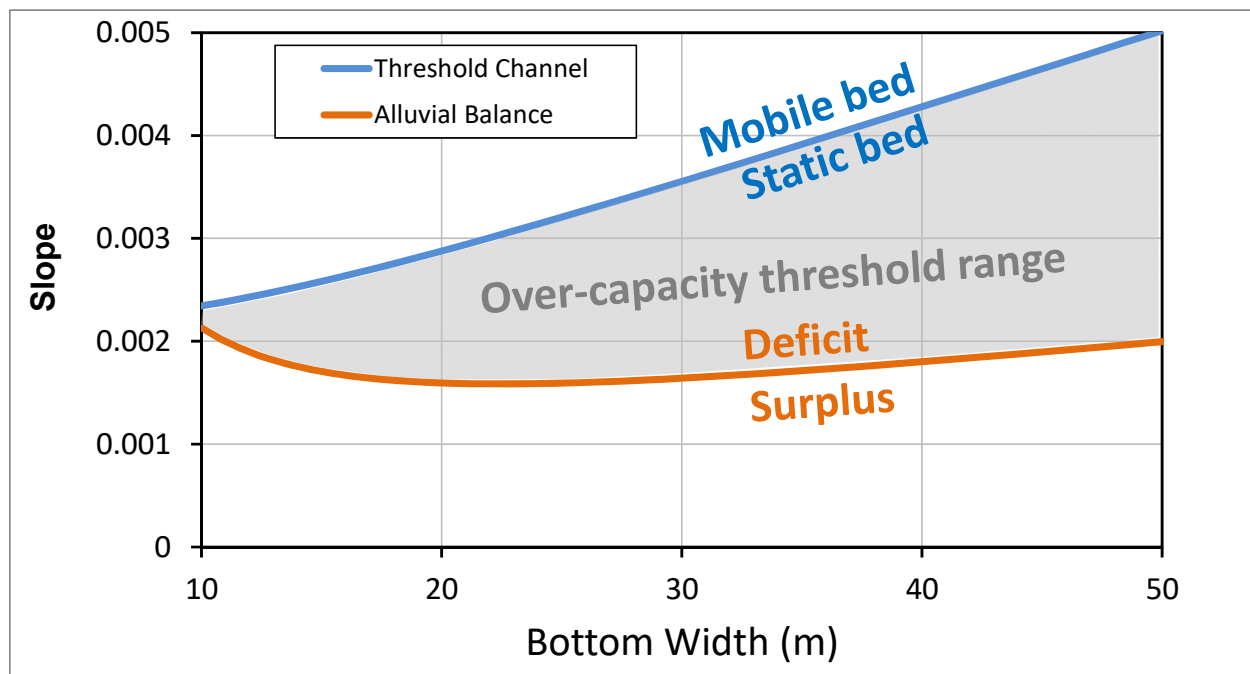
**Figure 5.** Threshold and mobile-bed solutions for the indicated channel, sediment and flow.

The over-capacity threshold condition is illustrated here to demonstrate that the range of transport behavior includes bed material mobility (threshold channel), sediment balance (mobile-bed channel), and a combination of threshold and mobile-bed behavior. Over-capacity threshold conditions are more common than often appreciated and are also called *semi-alluvial* (Ashmore and Church, 2001). Semi-alluvial channels are common in regions within the extent of continental glaciation, wherein lag deposits of larger glacial clasts can accumulate in river channels. Semi-alluvial channels are also found in steeplands with an active connection between hillslopes and river channels. Further, armoring and debris accumulation in incised urban streams may produce beds that are coarser than needed to transport the sediment supply.



## Risk and Uncertainty in Application

There is, of course, considerable uncertainty in the channel design process. Foremost among the uncertain terms is the future water and sediment supply. Uncertainty also arises from using relatively simple hydraulic and transport relations to represent actual transport and morphodynamic conditions. The risk posed by uncertainty can be illustrated using a diagram representing the solution for both threshold and alluvial channels. The case is illustrated for an over-capacity threshold channel such as in Figure 6. Channels with no over-capacity threshold range would show a threshold channel trend at slopes smaller than the mobile-bed trend. Related considerations are provided in Chapter 13 of NRCS (2007).



**Figure 6.** Sediment transport behavior relative to threshold and mobile-bed solutions.

Failure in a threshold channel occurs when the slope is too large, such that the bed is mobile at the design discharge. For an alluvial channel, a slope that is too small is in sediment surplus and will result in progressive sediment accumulation. A slope that is too steep is in sediment deficit and will experience progressive sediment evacuation if the flow has sufficient competence to move the bed material.

A channel may be intentionally designed to be over-capacity threshold by, for example, using large bed material. The bed is immobile or largely so at all flows while providing sufficient slope to transport the supplied sediment at all flows. In this case, the bed surface grain size and channel slope are not in equilibrium with the sediment supply. The transport capacity exceeds the supply, yet the channel is unable to adjust by evacuating bed material. Such designs have become common in urban streams in which the bed is anchored by large boulders. There is an engineering benefit to such a design if bed incision must be prevented to protect infrastructure. This is also an ecological cost to such a design in that the bed is static except for transient storage of supplied sediment. Eliminated is the potential for channel change and the development over time of a diverse, dynamic bed topography which may be desirable for a



thriving aquatic ecosystem. An over-capacity threshold design addresses uncertainty through the use of large bed material and a slope more than sufficient to transport the sediment supply.

Uncertainty associated with alluvial channel design – finding the graded slope that will transport the supplied sediment with the available flow – is more complex. Nonetheless, steps can be taken to accommodate the uncertainty based on the particulars of a given site. A useful strategy is to estimate the volume of sediment that might be stored or evacuated from the reach over a number of years. This returns to the initial standard: what is the supply of water and sediment and what to you want to do with it?

In a reach with an active sediment supply, if the design goal is to have a dynamic, healthy aquatic ecosystem, a suitable design choice could be to design the channel to store a fraction of its sediment supply, creating bars and a shifting channel that can create a diverse riparian habitat mosaic. This approach may be possible if sufficient space is available to allow for a shifting channel. The design slope and transport capacity would be based on an estimate of the magnitude of the sediment supply over all flows compared to the magnitude of stored sediment required to build key geomorphic features (e.g. a slope of 0.0012 in Figure 4). For example, for a typical channel bend, migration of the channel by one channel width would require storage of a quantifiable amount of sediment. If that amount of sediment storage is estimated to occur over a few years, one may well have the recipe for the desired, dynamic ecosystem.

In contrast, if a river is tightly constrained by infrastructure such as road crossings or utility lines, it could be important to design the channel close to the graded condition, such that there is neither sediment accumulation or evacuation. In this regard, it is useful to consider the variation in slope needed to transport a wide range of sediment supply rates (Figure 3). If the sediment supply rate is modest, the graded channel slope is relatively insensitive to sediment supply rate, such that a mismatch between the design slope and the true graded slope may involve minor or slow, thus acceptable scour or aggradation. In contrast, if the sediment supply rate is sufficiently large that the problem enters the range where the graded slope is sensitive to the sediment supply rate, additional effort to improve the estimate of the sediment supply rate may be warranted. In any case, an effort to estimate the actual volume of potential scour or deposition features compared to the magnitude of the sediment supply can be useful for informing design alternatives.

## Conclusions

River channel geometry is determined by the supply of water and sediment, mediated by the influence of vegetation and humans. When assessing or designing a stream channel, the essential questions are: “what is the supply of water and sediment and what do you wish to do with them?” (Wilcock, 1997). Channels may have little or no sediment supply and can be designed as threshold channels. Channels with non-negligible sediment supply can be designed to be in a graded state, or they may be designed to store or evacuate sediment. A common condition is one in which the channel bed is coarser than that needed to transport the supplied sediment, in which case the channel can be designed to be in sediment deficit, or over-capacity with respect to its sediment supply but also non-incising because of its coarsened river bed. Such a channel combines elements of both threshold and alluvial channels.

In none of these cases does channel design depend essentially on broad correlations of channel geometry as a function of discharge or drainage area. Once the relations between channel width and channel slope that produce the desired transport behavior have been determined, broad

correlations, as well as local observations of channel geometry, can be used to select channel width and subsequent dimensions. For a specified channel width, channel slope and depth are determined by the supply of water and sediment and the desired channel behavior. In none of these cases is a specific discharge required, whether dominant, effective, or two-year. Rather, the full range of discharge and its associated sediment supply are used to find the design slope.

A channel with banks will have a bankfull flow, of course. The selection of the bankfull discharge need not, and should not be made based on any broad correlation among channel dimension and various flow metrics. Not only are these relations based on self-adjusted channels that may have scant connection to a stream requiring attention, such an approach provides no direct linkage between the water and sediment drivers and desired attributes of channel behavior. The fallacy of this approach is immediately evident from changing fashion: over the years, the flow magnitude popular for channel design has decreased gradually, then suddenly, from order two-year flood to flows that occur every year to, in some cases, something closer to baseflow. Each stream channel must deal with the full range of water discharge and the associated sediment supply. So should the stream channel designer.

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