Web-mapping Application of Flow Runout for WOTUS Calculations

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Abstract

A critical impact of understanding the environmental impact of point water discharge is the extents and mapped delineation of its steady state runout, particularly since runout prediction determines if the flow reaches the Waters of the United States (WOTUS). To help with this assessment, the Arizona Department of Environmental Quality (ADEQ) requested that Matrix, with assistance from WEST, provide a web-mapping application that uses minimal, readily available parameters, to predict the runout from any point in Arizona. Matrix successfully developed this application, which first creates a base flow path by seamlessly combining multiple programs and programming languages, including Leaflet, Calcite Maps, Bootstrap, Esri ArcGIS API JavaScript, Turf.js, jQuery, and the USGS StreamStats Flow (Raindrop) Path algorithm. This overarching delineation is then truncated to a predicted runout path length determined by a closed-form equation developed by WEST. Both the runout application and runout length equation are believed to be the first of their kind.

Introduction

Since June of 2020, the Arizona Department of Environmental Quality (ADEQ) has worked to enforce the Clean Water Act (CWA) programs to the applicable watercourses within their responsibility. These watercourses may fall under the Navigable Waters Protection Rule (NWPR) which divides water bodies by CWA jurisdiction. Further complications arise from manmade outflows that discharge into jurisdictional waters, regardless of point source location. Under those circumstances, the entire watercourse, from point to outlet, will then be designated under CWA guidelines and thus those responsible for the point source will require a CWA permit.

Further regulations relate to Waters of the United States (WOTUS). WOTUS describes all major watercourses and waterbodies. Projects involving WOTUS are typically subjected to multiple requirements at different levels of government. Hence, understanding if and/or how a project discharge will impact a WOTUS becomes essential information. Hydraulic contact with a WOTUS triggers numerous site-specific requirements beyond the scope of this paper. The ADEQ (2023) WOTUS webpage (http://www.ADEQ.gov/wotus) and the NACO (2023) webpage (https://www.naco.org/resources/rewrite-waters-us-rule) both provide a good summary of the critical points, with references to other sources for more technical treatment.

Background and Goal

In response to the imperative need for methods to analyze runout from point sources to see if they reach WOTUS, the ADEQ procured a consultant team led by Matrix New World Engineering (Matrix) and supported by WEST Consultants, Inc. ADEQ provided clear instruction regarding the desired WOTUS assessment tool, which was to develop a tool to allow ADEQ staff and potentially permittees a simplified method by which to determine whether/under what conditions a given discharge could reach a WOTUS. Matrix, with assistance from WEST Consultants, divided this work into two separate efforts:

Runout Equation Development: First, WEST determined a defensible yet simple first principles derived equation for predicting runout. This equation uses readily available data sources including both project specific elements such as point discharge and the size of the contributing watershed, as well as natural aspects such as floodway width, 100-year discharge rates, and infiltration rates. The developed formula can thus be implemented utilizing information from the permittee, USGS StreamStats reports, USDA soils information, Arizona floodway information, and flow regime geospatial data. If necessary, other sources of data can be utilized in lieu of those listed above.

The final equation assumed uniform flow and a roughly parabolic shape (both a common assumption for natural channels) as well as constant values for the 100-year flow and infiltration (Equation 1). The full entire derivation is included as an appendix.

$$z_{runout} = S_f \frac{7q_{100}^{2/7}}{5p_{100}i} q_0^{5/7} \tag{1}$$

where

z_{runout}	=	Runout distance (ft)
q_{100}	=	100-year return flow estimate (cfs)
p_{100}	=	100-year return wetted perimeter estimate (ft)
q_0	=	Initial point source flow (cfs)
i	=	Infiltration rate (ft/sec)
S_f	=	Safety factor (unitless)

Web-mapping Application: Given the runout equation provided by WEST, Matrix developed a web-mapping application that allows ADEQ analysts to select a discharge point on a map, enter the parameters for the discharge, and then print a report outlining the path and length of the discharge. This web-based mapping application was built using Leaflet, Calcite Maps, Bootstrap, Esri ArcGIS API JavaScript, Turf.js, and jQuery. The USGS StreamStats Flow (Raindrop) Path application served as the basis to determine initial runout where no parameters are introduced. Once this path is determined, the analyst can use the Turf JavaScript editing tools to refine the runout path.

When the above methodologies were applied, several assumptions were challenged and required further consideration. These are:

1. The runout path is inevitably sinusoidal. The runout distance will be along the watercourse route and hence the Runout Tool must adjust accordingly.
2. Three of the constants in the runout equation depend upon average values across the runout length: The 100-year flood event Q_{100}, the corresponding wetted perimeter p_{100}, and the infiltration rate i. Hence the process is iterative: These variables are selected to estimate the total runout, and then revisited to better represent the average values

across-section runout distance. The computation is then run again. That said, the variation between iterations tends to be small by comparison with the overall length.

3. With the model established, three separate helpful and sometimes required computations can be made:
    a. The total expected runout (as described);
    b. The total distance to a WOTUS;
    c. The relationship between time and runout distance. This can be helpful for quasi-unsteady modeling with a known hydrograph. (The time relation aspect is shown in the appendix.)

Final Product

The final website, referred to as the Discharge Distance Determination (3D) website is shown in **Figure 1** below.

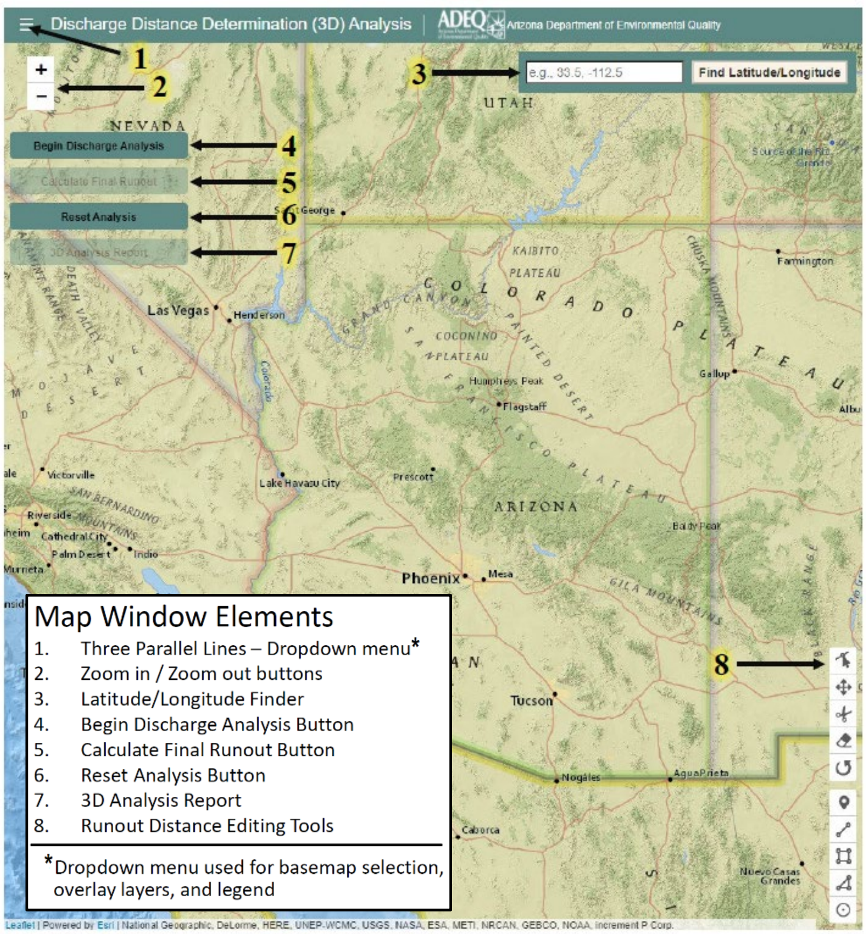

Figure 1. Discharge Distance Determination (3D) Analysis

The variety of user options is apparent, including zoom, select, pan, base map change, and other functions which should help verify the position of discharge point. The sequence of tasks required to obtain the ultimate runout computation using the 3D website are as follows:

1. Identify (assumed steady state) outflow from point source in gpm.
2. Determine the latitude and longitude (in decimal form) of the point source.
3. Determine the 100-year discharge (cfs) and drainage area (sq. mi.) at the point source. [StreamStats is one method for determining these values (Ries, et al. 2017). See lower right inset, **Figure 2**., which shows the StreamStats reported results.]
4. Input the coordinates into the 3D then press “Find Latitude and Longitude”. (See upper right input box, **Figure 1**.) Then press “Begin Discharge Analysis”.
5. Input the flow and area information into 3D. (See upper right input box, **Figure 4**.)
6. The program is ready to calculate. Press the “Calculate Final Runout” button.
7. The result is shown in **Figure 4** with numerical details shown as an inset. The predicted runoff is about 18 miles, just short of the French Creek tributary into Pleasant Lake.

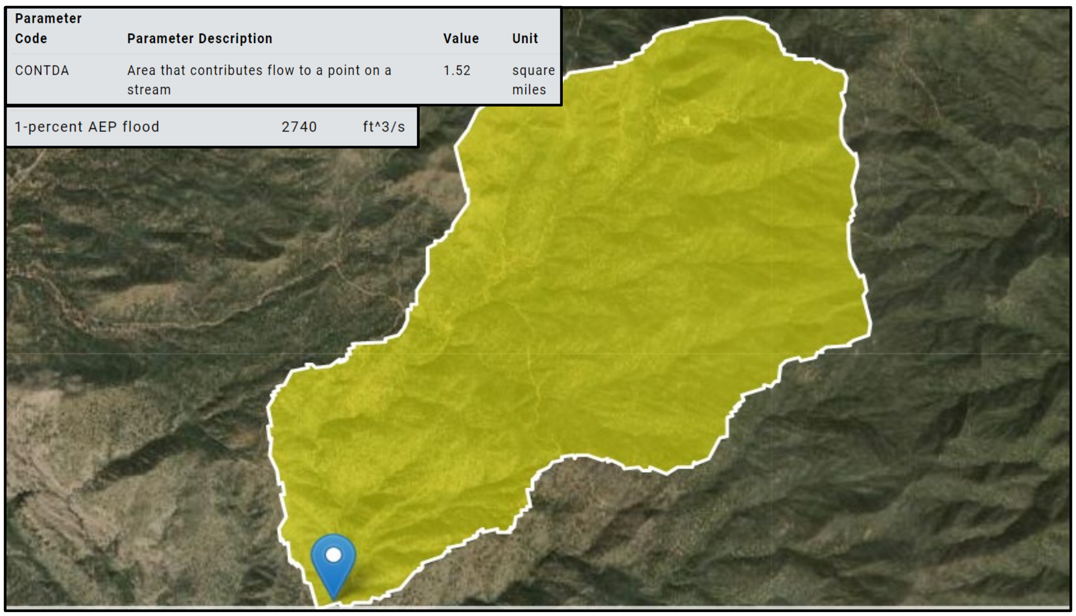

Figure 2. Results from StreamStats Hydrologic and Hydraulic Analysis

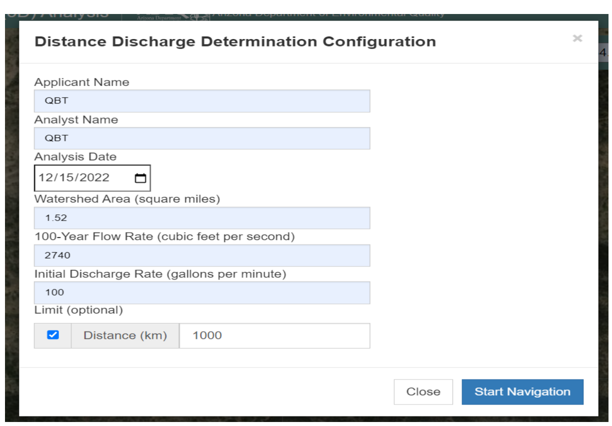

Figure 3. 3D Hydraulic and Hydrologic Data Input

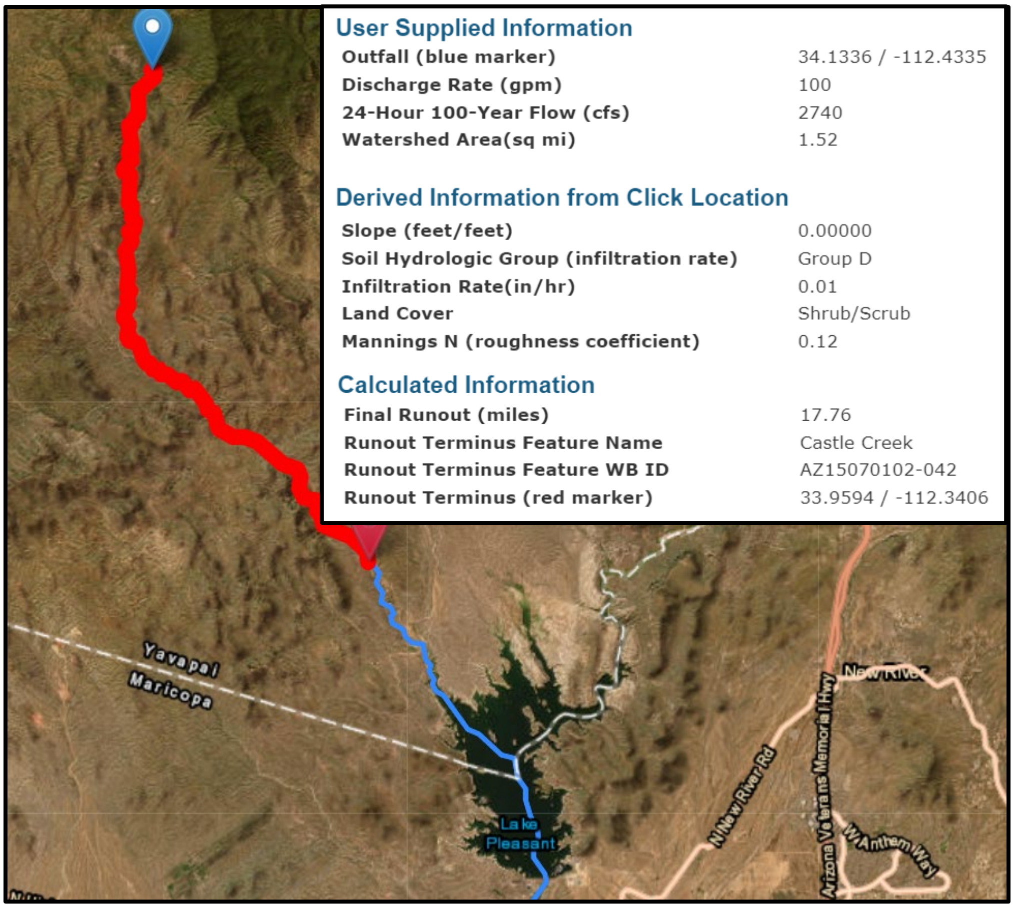

Figure 4. 3D Example Results

The example nondimensionalized runout geometry is also shown in **Figure 5**.

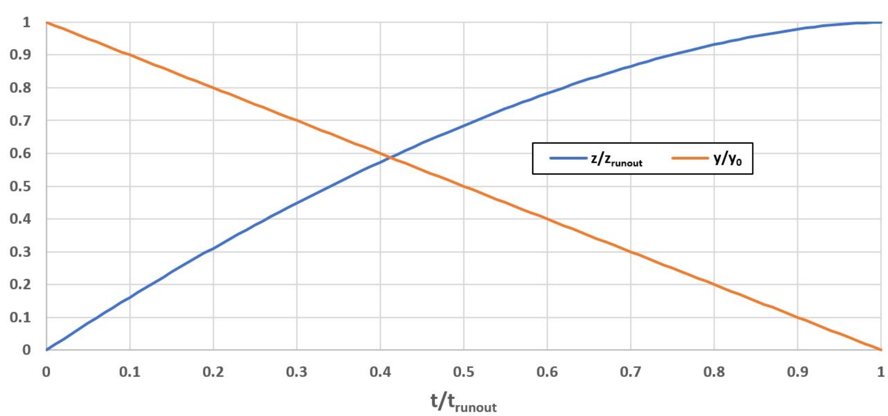

Figure 5. Example Runout Geometry

It is seen that the channel flow velocity gradually drops until reaching zero at the runout point. Note that the Manning's n is needed to describe how the depth changes over the runout distance but is not needed for the runout distance estimate itself.

Conclusion

It is an ongoing challenge to understand the impacts that manipulating our water supply has had and continues to have on human health, the environment, the water cycle, and nature as a whole. A key step in this learning process is to be able to accurately predict the threshold at which altered outflow enters back into the normal ecosystem. The process developed and described herein is a solid step in that regard, but there is still much that can and should be done. Examples include fully automating hydrologic prediction and developing guided web based hydraulic analysis tools.

Works Cited

Ries, K.G., III, J.K. Newson, M.J. Smith, J.D. Guthrie, P.A. Steeves, T.L. Haluska, K.R. Kolb, R.F. Thompson, R.D. Santoro, and H.W. Vraga. 2017. *StreamStats, version 4, Fact 2017–3046*. Washington, D.C.: U.S. Geological Survey.

Appendix A. Derivation of Runout Equations

This appendix provides details for the derivation of the proposed runout equation for ephemeral streams in Arizona. As corollaries to this derived equation, believed to be the first of its kind, equations describing the wetted perimeter, wetted area, and depth of flow can also be predicted along the runout reach.

Continuity

The derivation of the proposed equation begins with the continuity equation for one-dimensional flow:

$$\frac{dq}{dz}+\frac{da}{dt}=-ip \qquad (1)$$

where q (cfs) is the inflow at distance z (ft) from the point source q (cfs), and hence $q = q_o$ at $z = 0$, i (ft/sec) is the infiltration rate, a (ft^2) is the wetted area, and p (ft) is the wetted perimeter. Solving the continuity equation requires first relating the key cross-sectional parameters to a single variable [here the depth y (ft) is used] and then applying these relationships to the partial differential equation and solving. (Note that the boundary condition on y is that $y = y_0$ at $z = z_0$.)

Parameter Relationships

A natural cross-section can be reasonably approximated as a power equation relating depth y (ft) to horizontal distance x (ft), where $\underline{x}$ is equivalent to half the overall wetted width w (ft). This relationship may be written in general form as

$$\frac{y}{y_1}=\left(\frac{x}{x_1}\right)^{\kappa} \qquad x\geq 0 \qquad (2)$$

where x_1 (ft) and y_1 (ft) are known values at a particular flowrate q_1 (cfs), and κ is a dimensionless coefficient, typically varying between one and two for natural channels (Vatankhah 2015). Here, κ is assumed to be 3/2 - the average of the two extremes. As can be seen in Equation (2), with κ established at 3/2, any known values for x_1 and y_1 are suitable for establishing the power-law channel geometry. The 100-year flow values are good candidates in this regard. The 100-year flow q_{100} (cfs) and the 100-year floodway width w_{100} (ft) can be estimated using published equations as developed by the Arizona Department of Water Resources (ADWR) among others (note the "100" subscript is used here and subsequently to denote the given variable correspondence to the 100-year return event). Specifically, the two sources that provide a q_{100} estimate is ADWR Publication 2-96 (ADWR Flood Mitigation Section 1996) and StreamStats (Ries, et al. 2017) The 100-year floodway width w_{100} can also be estimated from ADWR Publication 2-96 (ADWR Flood Mitigation Section 1996) as

$$w_{100}=\eta A^{\lambda} \qquad (3)$$

where η and λ are determined by region as given in the referenced publication, and A is the watershed area in square miles. Of course, the floodway width is not equal to the floodplain width.

However here, for the purposes of the runout calculation, the floodplain width is assumed equal to the floodway width. The defense of this assumption is that the floodway width is always smaller than the floodplain. Hence, the assumption of their equality is conservative with regards to estimating runout length since the narrower the wetted perimeter (i.e., floodway width at the 100-year storm as compared with the floodplain), the slower the infiltration and longer the runout prediction.

Utilizing these published data allows the power-law relationship to be rewritten in terms of known 100-year depth and floodway width as follows:

$$\frac{y}{y_{100}} = \left(\frac{w}{w_{100}} \right)^{3/2} \tag{4}$$

Please note that while the 100-year value for x has been established in terms of the published half width x_{100} (i.e., $x_{100} = \frac{1}{2} \times w_{100}$), the depth of flow y_{100} (ft) has not been specified. This value will be determined subsequently as a byproduct of deriving the runout equation.

It must be stressed that the 100-year return event values shown in Equation (4) and subsequently are included purely as a means to establish the cross-section geometry. They are given values. They cannot be determined by the derived equations, nor are they used directly to compute runout, but rather indirectly to help establish to bounds of the runout equation. They can be replaced by any set of known values at a particular flow.

The hydraulics component will require estimates of the wetted perimeter p (ft) and wetted area a (ft^2). Using x as the dummy variable for the integration over width, the wetted perimeter for the power-law section can be determined by evaluating

$$p = 2 \int_0^{w/2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \, dx \tag{5}$$

which results in the following closed form equation for the wetted perimeter:

$$p = \frac{2 w_{100}^3}{27 y_{100}^2} \left[\left(1 + 9 \frac{y_{100}^2}{w_{100}^3} w \right)^{3/2} - 1 \right] \tag{6}$$

Despite the closed form solution for p given above, it's complexity ultimately prevents a closed form solution for the runout estimate. To obtain an explicit equation for the runout, it is noted that the wetted perimeter will always be greater than the wetted width. Hence, assuming $p = w$ is conservative with regards to runout prediction since it will underpredict the total infiltration rate. (Note that the assumption of $p = w$ in general implies that $p_{100} = w_{100}$ specifically.) Given this assumption, the relationship between depth and wetted perimeter can be taken directly from Equation (4) as follows:

$$\frac{y}{y_{100}} = \left(\frac{p}{p_{100}} \right)^{3/2} \tag{7}$$

Again, it is noted that p and p_{100} are assumed equal to the wetted widths w and w_{100} respectively. Solving for y in Equation (7) thus results in the simple equation for y in terms of p:

$$y = y_{100} p_{100}^{-3/2} p^{3/2} \tag{8}$$

Equation (8) can also be inversed to obtain the perimeter p as a function of y:

$$p = p_{100} y_{100}^{-2/3} y^{2/3} \tag{9}$$

The next step in the derivation is to develop equations for the wetted area. The wetted area of the assumed power-law representation of the natural channel is expressed as the following integral:

$$a = y_{100} p_{100}^{-3/2} p^{5/2} - 2^{5/2} y_{100} p_{100}^{-3/2} \int_0^{p/2} x^{3/2} dx \tag{10}$$

The solution of Equation (10) is

$$a = \alpha y_{100} p_{100}^{-3/2} p^{5/2} \tag{11}$$

where the constant α is[1]

$$\alpha \equiv 3/5 \tag{12}$$

It is simple yet useful to find a_{100} is thus from Equation (11):

$$a_{100} = \alpha y_{100} p_{100} \tag{13}$$

The wetted area a can be expressed in terms of y expressed as a function of y is then found by replacing p in Equation (11) with its value in terms of y given in Equation (9). The result is

$$a = \alpha p_{100} y_{100}^{-2/3} y^{5/3} \tag{14}$$

The preceding equations relate the hydraulic parameters but do not consider the flowrates q_{100} and q_o, both needed to obtain an estimate of y_{100}, y_o, and eventually the runoff distance and time to reach final runout. To that end, Manning's equation can be used to establish this relationship. Manning's equation in general form is

$$q = \phi n^{-1} a^{5/3} p^{-2/3} s^{1/2} \tag{15}$$

where φ is the unit conversion constant (1.49 for the English units used here, 1.0 for SI units), n (dimensionless) is the channel roughness coefficient, and s (ft/ft) is the frictional slope, typically assumed to be the longitudinal slope. Equation (15) can be written as a function of depth y through substitution of Equation (14) for a and Equation (9). The result is

$$q = \phi \alpha^{5/3} s^{1/2} n^{-1} p_{100} y_{100}^{-2/3} y^{7/3} \tag{16}$$

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1 Note that the original report reported the incorrect value for α. The corrected value is shown here. This error has no impact on the final results.

Note that for the 100-year storm, the corresponding predicted flow in Equation (16) is

$$q_{100} = \phi\alpha^{5/3} s^{1/2} n^{-1} p_{100} y_{100}^{5/3} \tag{17}$$

Both the 100-year flow q_{100} and the 100-year flow wetted perimeter p_{100} are known values, and hence y_{100} can now be estimated explicitly in terms of the known values q_{100} and p_{100} by inverting Equation (17) to obtain

$$y_{100} = \phi^{-3/5} \alpha^{-1} s^{-3/10} n^{3/5} p_{100}^{-3/5} q_{100}^{3/5} \tag{18}$$

Substituting y_{100} into the previous explicit equations developed for the hydraulic variables a, p, and q (i.e., Equations (18), (9), and (14)], the following 3 explicit equations can be established, each dependent upon only given site parameters:

$$a = \phi^{2/5} \alpha^{5/3} s^{1/5} n^{-2/5} p_{100}^{7/5} q_{100}^{-2/5} y^{5/3} \tag{19}$$

$$p = \phi^{2/5} \alpha^{2/3} s^{1/5} n^{-2/5} p_{100}^{7/5} q_{100}^{-2/5} y^{2/3} \tag{20}$$

$$q = \phi^{7/5} \alpha^{7/3} s^{7/10} n^{-7/5} p_{100}^{7/5} q_{100}^{-2/5} y^{7/3} \tag{21}$$

As noted previously, the boundary conditions for these equations are $q = q_0$ at $z = 0$ and $y = y_0$ (ft) at $z = 0$. The initial depth y_0 can be established by applying Equation (21) at the outlet:

$$q_0 = \phi^{7/5} \alpha^{7/3} s^{7/10} n^{-7/5} p_{100}^{7/5} q_{100}^{-2/5} y_0^{7/3} \tag{22}$$

which can be inverted to obtain an explicit solution for y_0:

$$y_0 = \phi^{-3/5} \alpha^{-1} s^{-3/10} n^{3/5} p_{100}^{-3/5} q_{100}^{6/35} q_0^{3/7} \tag{23}$$

Continuity Equation Solution

The derived hydraulic parameters permit solution of the governing continuity equation. The required derivations are straightforward application of the differentiation power-laws. The differentiation terms for a and q can be directly evaluated from Equations (19) and (21) respectively:

$$\frac{\partial a}{\partial t} = \tfrac{5}{3} \phi^{2/5} \alpha^{5/3} s^{1/5} n^{-2/5} p_{100}^{7/5} q_{100}^{-2/5} y^{2/3} \frac{\partial y}{\partial t} \tag{24}$$

$$\frac{\partial q}{\partial z} = \tfrac{7}{3} \phi^{7/5} \alpha^{7/3} s^{7/10} n^{-7/5} p_{100}^{7/5} q_{100}^{-2/5} y^{4/3} \frac{\partial y}{\partial z} \tag{25}$$

The equation for the wetted perimeter p as a function of y is given explicitly by Equation (20). Hence, the continuity equation can now be expressed in terms of depth by using equations (20), (24), and (25) to obtain

$$\begin{aligned}&\tfrac{7}{3}\phi^{7/5}\alpha^{7/3}s^{7/10}n^{-7/5}p_{100}^{7/5}q_{100}^{-2/5}y^{4/3}\frac{\partial y}{\partial z}+\tfrac{5}{3}\phi^{2/5}\alpha^{5/3}s^{1/5}n^{-2/5}p_{100}^{7/5}q_{100}^{-2/5}y^{2/3}\frac{\partial y}{\partial t}\\&=-i\phi^{2/5}\alpha^{2/3}s^{1/5}n^{-2/5}p_{100}^{7/5}q_{100}^{-2/5}y^{2/3}\end{aligned} \tag{26}$$

Simplifying, this equation can be expressed as

$$\tfrac{7}{5}\phi\alpha^{2/3}s^{1/2}n^{-1}y^{2/3}\frac{\partial y}{\partial z}+\frac{\partial y}{\partial t}=-\tfrac{3}{5}i\alpha^{-1} \tag{27}$$

Which is directly solvable using kinematics as follows:

It is first observed that the complete derivative (as opposed to partial derivative) of y is defined by

$$dy=\frac{\partial y}{\partial z}dz+\frac{\partial y}{\partial t}dt \tag{28}$$

Dividing by dt and reversing the equality results in the equation

$$\frac{dz}{dt}\frac{\partial y}{\partial z}+\frac{\partial y}{\partial t}=\frac{dy}{dt} \tag{29}$$

Note that Equation (27) is identical to (29) provided:

$$\frac{dy}{dt}=-\tfrac{3}{5}i\alpha^{-1} \tag{30}$$

and

$$\frac{dz}{dt}=\tfrac{7}{5}\phi\alpha^{2/3}s^{1/2}n^{-1}y^{2/3} \tag{31}$$

Using the boundary condition that $y = y_0$ at $z = 0$, the solution to Equation (30) is

$$y=y_0-\tfrac{3}{5}i\alpha^{-1}t \tag{32}$$

Using the full equation for y_0 given in Equation (23), this equation becomes:

$$y=\phi^{-3/5}\alpha^{-1}s^{-3/10}n^{3/5}p_{100}^{-3/5}q_{100}^{6/35}q_0^{3/7}-\tfrac{3}{5}i\alpha^{-1}t \tag{33}$$

The full runout time t_{runout} (sec) occurs when y in equation (33) equals zero. That is, at time t_{runout} where

$$t_{runout}=\tfrac{5}{3}\phi^{-3/5}s^{-3/10}i^{-1}n^{3/5}p_{100}^{-3/5}q_{100}^{6/35}q_0^{3/7} \tag{34}$$

To obtain z_{runout}, it is noted that given Equation (32), Equation (31) can be rewritten as:

$$\frac{dz}{dt} = \tfrac{7}{5}\phi s^{1/2} n^{-1}\left(\phi^{-3/5} s^{-3/10} n^{3/5} p_{100}^{-3/5} q_{100}^{6/35} q_0^{3/7} - \tfrac{3}{5} it\right)^{2/3} \quad (35)$$

which has the solution

$$z = \tfrac{7}{5} i^{-1} p_{100}^{-1} q_{100}^{2/7} q_0^{5/7} - \tfrac{7}{5}\phi s^{1/2} i^{-1} n^{-1}\left(\phi^{-3/5} s^{-3/10} n^{3/5} p_{100}^{-3/5} q_{100}^{6/35} q_0^{3/7} - \tfrac{3}{5} it\right)^{5/3} \quad (36)$$

The total runout occurs when parenthesis term in Equation (36) equals 0 (i.e., when $t = t_{runout}$), as shown in Equation (34). The result is simply

$$z_{runout} = \frac{7 q_{100}^{2/7}}{5 p_{100} i} q_0^{5/7} \quad (37)$$

It is important and somewhat surprising that the total runout distance is not a direct function of Manning's roughness coefficient nor the longitudinal slope s. It appears that these hydraulic values are inferred by q_{100} and p_{100} to which n and s are both implicitly related.

Conclusions

Each term in Equation (37) is consistent with expectations:

1. As the 100-year flow q_{100} increases, so does z_{runout}. In general, a higher 100-year flow is an indicator of a relatively steeper channel, lower roughness, and potentially relatively narrower floodplain (thus improving hydraulic efficiency), or some combination of the three, all of which would be expected to extend runout distance.
2. The runout distance is predicted to increase with the input flow q_0. This is as expected – a higher flow in the system will require a longer distance to completely infiltrate.
3. As the 100-year floodway width p_{100} increases, the overall runout length z_{runout} is predicted to decrease. This is consistent with the general observation that an increased p_{100} will establish a wider floodway at a given flow rate.
4. As expected, the infiltration decreases the runout length at higher values and increases it at lower values.

The constant values used for this derivation are inherently uncertain. The 100-year flow, 100-year wetted perimeter, are both highly uncertain. The assumption of a parabolic cross-section, a well vetted assumption, is unlikely to adequately capture all watercourses. Field measurement of the infiltration rate can be very challenging. Further, the cross-section geometry and infiltration rate are unlikely to remain unchanged throughout the length of the runoff distance.

Unfortunately, it is difficult to quantify the above noted uncertainties. However, the derived equations provide the best estimate. To account for these uncertainties, it is recommended that a safety factor (S_f, dimensionless) be applied. The value of this safety factor should be set by qualified project personnel. Hence, the final recommended runout equation is

$$z_{runout} = s_f \frac{7q_{100}^{2/7}}{5p_{100}i} q_0^{5/7} \tag{38}$$

where $s_f > 1$.

Works Cited

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