# Erosion Countermeasure Design in Complex Flow Field: A Case Study on the American River

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## Abstract

The American River Common Features project in Sacramento California identified a portion of a levee on the North side of the floodplain in the vicinity of multiple bridges requiring erosion protection to reduce flood risk. This is needed to protect the levee from high velocities and turbulent flow around the bridge features. Some of the columns, posts, and abutments of these bridges are near the levee, increasing turbulence near the levee. Understanding the erosion risk to the levee, including effects of flow through and around the bridge structures, is needed to minimize environmental impacts and reduce cost while still protecting the large at-risk population living behind the levee.

The risk-informed design at this site utilized the Bank Stability and Toe Erosion Model (BSTEM) developed by USDA-ARS to evaluate bank retreat. The results from the BSTEM analysis were used to inform the initiation and progression nodes of the erosion event tree used for the risk assessment. Consideration was given to the development of scour around the bridges near the levee toe. The levee toe was found to be outside the zone of increased scour from flow around the bridge piers. A Monte-Carlo simulation was evaluated within BSTEM to provide quantitative bank retreat estimates to inform the existing and proposed project risk of an erosion related levee breach at the site. Countermeasures considered included stone protection and constructing a berm upstream of the at-risk area to reduce velocities. BSTEM model scenarios were developed for the existing conditions and proposed countermeasures to inform the pre-and post-construction risk assessment. The collective BSTEM results along with all other available information was used successfully to inform the risk assessment used for the risk-informed design of erosion countermeasures for the levee near bridges with complex flow to optimize the design to reduce life safety risk.

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## Introduction

The American River Common Features project in Sacramento California identified a portion of a levee on the North side of the American River floodplain in the vicinity of multiple bridges as needing erosion protection to reduce flood risk for the city of Sacramento. Three bridges span the Lower American River (LAR) in this area, denoted as LAR 2.0R. The embankments of the bridges obstruct floodplain flow (Figure 1) leading to higher velocities near the right levee. The presence of the bridge bents near the levee toe creates turbulent flow that can lead to scour and erosion into the levee and along the levee toe. In addition, the embankment supporting the railroad bridge protrudes into the flow causing additional flow constriction in this area.

Soil investigations indicate the soil is mostly low plasticity silts (Unified Soil Classification System - USCS, ML) and clays (USCS, CL) with relatively low blow counts (Figure 2). Erosion Function Apparatus (EFA) and Jet Erosion Test (JET) results indicate the soils are categorized as high to medium erodibility (NASEM 2019) consistent with results of previous nearby tests of similar soils. The combination of high velocities, turbulent flow, erodibility of the soil coupled with a large at-risk population made this portion of the levee high risk. Recent guidance from the U.S. Army Corps of Engineers requires implementation of risk-informed design for new levee improvements and countermeasures. Risk assessments help minimize environmental impacts and cost while keeping life-safety risk to tolerable levels.



**Figure 1.** Site LAR 2.0R project vicinity map with contours and 2D HEC-RAS model velocities for 160,000 cfs discharge showing bridge embankments obstructing floodplain flow and increasing velocity through the bridge opening (approximately area of the pink rectangle)



Figure 2. Three closest borings to the BSTEM sites with approximate BSTEM soil layers overlayed

## Methods

### **Risk-informed Design and Risk Assessments**

Risk assessments are the cornerstone for risk-informed design (USACE 2019). A risk assessment following standard USACE guidelines was conducted that includes the existing (baseline) conditions and two proposed ("with-project") designs to lower levee erosion risk under the bridges. One design utilized rock to armor the levee and levee toe (stone protection design) in accordance with FHWA HEC-23 (FHWA 2009) and EM 1110-2-1601 (USACE 1994) guidelines. The other design relies on reducing the velocity against the levee by placing a berm upstream of the bridges (berm design) to serve as a deflector spur (FHWA 2009, Design Guide 2). The risk assessments focused on the probability of breach prior to overtopping the levee system as the consequences had been determined previously.

### **BSTEM Modelling for Probabilistic Levee Erosion Estimates**

To help inform the project's risk-informed design assessments, estimates of levee erosion at two cross-sections using BSTEM were provided to the risk cadre. The two cross-section locations, LAR 1.98R and LAR 2.16R (Figure 2) were selected due to the presence of high velocities and bridge bents close to the levee toe. BSTEM (USDA 2019) was selected as it models fluvial erosion and bank mass-wasting failures interactively, it can use inputs from 2D hydraulic models, and it can perform Monte-Carlo simulation of uncertain input parameters to provide probabilistic bank retreat estimates. BSTEM uses the linear excess shear equation to compute the erosion rate for estimating fluvial erosion. See Simon et al. (2011) for additional information on the BSTEM model.

Two of the most sensitive parameters in BSTEM are the erodibility coefficient and critical shear stress. BSTEM stochastic model runs were used to sample the distribution of these parameters

for Monte-Carlo computations to provide probabilistic bank retreat estimates for the risk assessments.

The BSTEM model erosion estimates inform expert opinion elicitation of the probability of a levee breach due to erosion. The BSTEM model erosion estimates provide computational information for the risk cadre to consider when estimating the probability of erosion initiation and progression in the erosion related levee breach event tree. The erosion levee breach event tree includes the following:

- 1) <u>Loading.</u> Flood loading.
- 2) <u>Flaw.</u> Failure of surface protection.
- 3) <u>Initiation.</u> Erosion initiation.
- 4) <u>Progression.</u> Erosion progression into the levee prism or its foundation.
- 5) <u>Detection and Intervention</u>. Detecting erosion and intervening (i.e., "flood fighting").
- 6) <u>Breach.</u> Breach of the levee crest causing uncontrolled water release into the protected area.

BSTEM does not provide information on all the nodes in the event tree. Therefore, the risk cadre utilizes other information besides the BSTEM erosion estimates to determine the probability of an erosion related levee breach prior to levee overtopping.

BSTEM models for existing conditions and with-project conditions were developed to inform the risk assessment. Existing condition models are used to inform the baseline risk assessment that determines if the site meets project risk objectives. With-project BSTEM models are used to inform the with-project risk assessment that determines if the site meets risk objectives with the proposed design constructed. The two with-project conditions are the berm (Figure 3) and stone protection (Figure 4) designs.



Figure 3. Berm design upstream of highway and railroad bridges showing LAR 1.98R and LAR 2.16R BSTEM crosssection locations



Figure 4. Stone protection design cross-section immediately downstream of LAR 2.16R

For the stone protection design, BSTEM is used to estimate potential erosion above the top of the rock to inform the risk assessment. HEC-23 guidelines (FHWA 2009) require rock be placed nearly to the top of the levee slope to an elevation of the incipient overtopping of the levee system (40 ft NAVD 88 vertical datum) or higher. However, EM 1110-2-1601 indicates that it may be possible for the top of rock to be placed to a lower elevation. A lower top of rock elevation can lower cost and simplify construction details near the bridges. BSTEM modeling of the stone protection design with the top of rock at elevation 34 ft (NAVD 88 vertical datum) is used to inform the risk assessment to determine if this stone protection design meets the project's risk reduction objectives. Similarly, BSTEM modeling of the berm design is used to determine if this design meets project risk reduction objectives.

#### **BSTEM Hydraulic Inputs**

A calibrated 2D Unsteady HEC-RAS model with 20-foot mesh (5-foot mesh refinement region near the project site) was developed for this site using a hydrograph with a peak discharge of 192,000 cfs. This is the discharge that floodwaters overtop the levee system. The 2D model shear stress computed at the toe of the levee was used as a direct input into the unsteady BSTEM models. Hydraulic models were developed for existing ground and with the stone protection and berm countermeasures.

To account for the possible additional shear stress from flow around the piers, the team considered using an equation that estimates the maximum shear stress at the pier (FHWA 2012, equation 7.36) using the velocity from the 2D HEC-RAS model. However, the effect of the pier will lessen until there is no additional shear stress from the pier at the outer edge of the maximum width of the scour hole. The computed scour hole depths and widths (assuming scour hole side slopes are 2H:1V) at this site indicates that the effects of the pier will be effectively eliminated at the face of the levee. Therefore, additional shear stress from flow around the piers is not needed at this site.

The Manning's n value used in BSTEM is used to compute the grain shear stress applied to the soil, which includes the effects of vegetation cover. This is a different application then the Manning's n values used in hydraulic modeling. The Manning's n values used in the BSTEM model were selected based on a comparison of land covers from nearby previous calibrated BSTEM models and a value of 0.035 was selected.

#### **BSTEM Geometry**

BSTEM Geometry was developed using the HEC-RAS 2D model terrain. This surface was developed using combined data from LiDAR, single-beam sonar, and topographic surveys.

Two cross-sections for BSTEM model development were selected. One includes the railroad bridge where its bridge bents are near the State Highway 160 bridge columns and the levee. This is denoted as LAR 1.98R. The other site is located just upstream of the State Highway 160 Eastbound bridge and is denoted as LAR 2.16R. This cross-section is selected to represent the conditions just upstream of the bridges. These are shown in Figure 1. The levee geometry, expected soils, and hydraulics do not vary substantially over the short distance downstream of LAR 2.16R to the State Highway 160 Eastbound bridge columns. Therefore, a BSTEM model geometry developed just upstream of the bridges is modified to represent scour conditions at the State Highway 160 Eastbound bridge columns.

BSTEM does not model vertical scour interactively. To account for this, scour estimates were computed and used to adjust the starting geometry of the BSTEM models. The scour estimates were obtained using HEC-18 (FHWA 2012) equations for cohesive soils. Contraction, pier, and abutment scour estimates were computed for bridge features near the toe of the levee slope. The scour equations were developed for design application and are intended to ensure very low probability that actual scour will exceed the computed value. The probabilistic bank retreat estimates from BSTEM using the geometry with and without scour therefore bounds the expected values if BSTEM could incorporate scour directly in the model. BSTEM model geometry for the berm alternative is unchanged from existing conditions as the berm is constructed upstream of the analyzed cross-sections.

#### **BSTEM Soil Inputs**

Five soil layers were developed for the BSTEM models using predominate soils shown in existing boring logs within 1,000 feet upstream and downstream of the site with more weight given to the borings closest to the modeling site. The soil layer thicknesses were selected from the boring log information with consideration of the bank geometry and stone protection design. The approximate soil layers are shown graphically over the 3 nearest borings to the BSTEM locations in Figure 2.

The soil parameters for each soil layer were selected from a set of calibrated parameters that had been compiled from seven previously calibrated BSTEM models from different site locations on the American River (Table 1). The method used to develop these parameters is documented elsewhere (Rivas et al. 2021a, Rivas et al. 2021b). The soil parameters are shown in Table 1. The soil parameters used for the stochastic modeling are shown in Table 2. For the stochastic models, a triangular distribution was used for the critical shear stress and erodibility coefficient parameters for all of the soil types. A triangular distribution was used for the stochastic parameters based on the results of previous work (Rivas et al. 2021a, Rivas et al. 2021b). For the stochastic models for stone protection, the  $D_{50}$  was assumed to be the median value for the triangular distribution.  $D_{10}$  was assumed to be the smallest riprap size and  $D_{100}$  was assumed to be the largest riprap size. The minimum and maximum values of the riprap were used to calculate the triangular peak (parameter a). For stochastic modeling, only the erodibility coefficient and critical shear stress were varied. All other parameters shown in Table 1 remain the same.

#### Table 1. BSTEM Soil Parameters

Layer	Soil type	Friction angle (degrees)	Cohesion (Ibf/ft2)	Saturated unit weight (Ibf/ft3)	Suction angle (degrees)	Hydraulic conductivity (ft/d)	Critical shear stress (lb/ft2)	Erodibility coefficient (ft3/lbf-hr)	Manning's n
1	CL	31.3	100.0	118.9	10.0	0.01	0.277	0.0450	0.035
2	ML	31.5	0.7	113.0	10.0	0.10	0.205	10.0000	0.035
3	ML	31.5	0.7	113.0	10.0	0.10	0.205	10.0000	0.035
3	SM	28.7	0.0	120.4	10.0	0.70	0.160	7.0000	0.035
4	ML	31.5	0.7	113.0	10.0	0.10	0.205	10.0000	0.035

#### Table 2. Soil parameters used for stochastic BSTEM modeling

Soil type: Clay (CL)	Distribution	Calibrated Value	Minimum Value	Maximum Value	Parameter a
Critical Shear Stress (psf)	Triangular	0.277	0.002	0.880	0.052
Erodibility Coefficient (ft3/lbf-hr)	Triangular	0.045	0.010	0.086	0.042
Soil type: Silt (MH/ML)	Distribution	Calibrated Value	Minimum Value	Maximum Value	Parameter a
Critical Shear Stress (psf)	Triangular	0.205	0.002	0.650	0.039
Erodibility Coefficient (ft3/lbf-hr)	Triangular	10.000	0.350	59.600	0.350
Soil type: Silty sand (SM)	Distribution	Calibrated Value	Minimum Value	Maximum Value	Parameter a
Critical Shear Stress (psf)	Triangular	0.160	0.003	0.650	0.003
Erodibility Coefficient (ft3/lbf-hr)	Triangular	7.000	0.350	40.130	0.350
Soil type: Riprap (RR)	Distribution	Calibrated Value	Minimum Value	Maximum Value	Parameter a
Critical Shear Stress (psf)	Triangular	6.490	1.397	10.313	7.230
Erodibility Coefficient (ft3/lbf-hr)	Triangular	0.0064	0.005	0.014	0.00107

### Results

#### **Existing Conditions**

**LAR 1.98R:** The probabilistic BSTEM results for LAR 1.98R are shown below for without scour (Figure 5) and with scour (Figure 6). The plots show a minimum theoretical levee and levee foundation template with a 20' top width at the elevation of the overtopping discharge with 3H:1V waterside slope and 2H:1V landside slope. This minimum theoretical levee template can be helpful for assessing levee erosion risk. The percentiles in the stochastic plots are nonexceedance values. Figure 5 implies that the likelihood of erosion initiation and progression into the levee or levee foundation (shown as a dotted line) is less than or equal to about 1 percent. Figure 6 indicates that the computed scour already encroaches into the theoretical levee foundation which is the starting condition for the BSTEM model. Figure 6 indicates the likelihood of further lateral erosion initiating and progressing further into the levee template after the maximum scour hole develops is about 10 percent. However, since the scour occurs first in Figure 6, an additional node before progression for vertical scour is likely needed on the event tree. This node would be the probability that the computed scour depth occurs, which being a conservative design equation is relatively low. Therefore, even though Figure 6 indicates a higher likelihood of a levee breach with scour, an added node to the event tree to account for scour occurring before lateral erosion will reduce the likelihood further.



Figure 5. LAR 1.98R existing conditions without scour probabilistic BSTEM results



Figure 6. LAR 1.98R existing conditions with scour

While not a complete probability of levee breach estimate from expert opinion elicitation, these results indicate there is likely a low likelihood of a levee breach at this location for existing conditions. The landside toe elevation shown on these plots stops at station -29 but the actual

maximum difference between the water surface elevation and the low point on the waterside of the levee (not shown on the plot) is about 1.5 feet. Therefore, even if a breach were to occur, the depth of the water flowing through the breach would be relatively low.

These findings are consistent with discussions during the expert opinion elicitation. The evidence suggests that erosion countermeasures likely are not needed at this cross-section.

**LAR 2.16R:** The probabilistic BSTEM results for LAR 2.16 are shown below for without scour (Figure 7) and with scour (Figure 8). Figure 7 implies that the likelihood of erosion initiating and progressing into the levee or levee foundation (shown as a dotted line) is about less than or equal to 25 percent, which is higher than at LAR 1.98R. Figure 8 indicates that the computed scour already encroaches into the theoretical levee foundation which is the starting condition for the BSTEM model. Figure 8 indicates the likelihood of further lateral erosion initiating and progressing further into the levee template after the maximum scour hole develops is about 25 percent, which is higher than for LAR 1.98R. Similar to LAR 1.98R, since the scour occurs first, an additional node before progression for vertical scour is likely needed on the event tree and this additional node would likely have the effect of decreasing the overall probability of levee breach since the computed scour is from a design equation and the computed scour is less likely to occur. While not a complete probability of levee breach estimate from expert opinion elicitation, these results indicate there is a relatively higher likelihood of a levee breach at this location for existing conditions compared to LAR 1.98R. The landside elevation is also much lower at this location and the levee width narrower. These findings are consistent with the results of the expert opinion elicitation, which consider other factors as well, which indicate this site does not meet the risk objectives for the project and needs erosion countermeasures.



Figure 7. LAR 2.16R existing conditions without scour



Figure 8. LAR 2.16R existing conditions with scour

#### **Stone Protection Design**

A proposed design that places rock along the toe of the levee and the levee slope to an elevation of 34 ft (NAVD 88) was modeled in BSTEM to inform the risk assessment for the design. In particular, understanding the risk of a levee breach for the lower top of rock elevation compared to FHWA guidelines is needed. The velocities are high enough (up to 4 fps in locations) to potentially initiate erosion, but the likelihood of progression leading to a levee breach from these higher velocities is not known. BSTEM modeling of the stone protection design was completed to better understand the likelihood of progression and inform the risk assessment.

**LAR 1.98R:** The results of the BSTEM modeling for LAR 1.98R are shown in Figure 9. For LAR 1.98R, the BSTEM results indicate a very low likelihood of erosion initiating or progressing above the top of rock (less than 1 percent).

**LAR 2.16R:** The results at LAR 2.16R (Figure 10) indicate erosion initiating and progressing into the levee above the top of rock may have a likelihood in the range of 10 - 25 percent. This is lower than without the stone protection but is still elevated. The risk assessment indicated that the repair likely does meet the risk objectives for the project, but the risk cadre recommended reviewing the top of rock elevation and consider increasing its height at this location to increase confidence and performance of the design.

#### **Berm Design**

A proposed design that places a berm upstream to reduce velocities against the levee was modeled in BSTEM to inform the risk assessment for the design. The berm BSTEM model uses hydrodynamics extracted from a revised 2D model that incorporates the proposed berm design. In particular, BSTEM modeling of the berm design was completed to better understand the likelihood of progression. The risk assessment for the berm design has not yet been completed.



Figure 9. LAR 1.98R Stone protection design stochastic BSTEM results



Figure 10. LAR 2.16R Stone protection design stochastic BSTEM results

**LAR 1.98R:** The results of the BSTEM modeling for LAR 1.98R are shown in Figure 11. For LAR 1.98R, the BSTEM results indicate a very low likelihood of erosion initiating or progressing into the levee or levee foundation (between about 1 to 5 percent). This is slightly higher than existing conditions but still low. Comparing the applied shear stress at the toe of the levee for the existing condition and the berm design indicates increased shear stress is the cause. However,

the magnitude of the shear stress is still below the critical shear stress values for CL and ML in Table 1. Therefore, the increased shear results and the resulting increased erosion shown in the BSTEM plot for the berm compared to existing conditions is not substantial.

**LAR 2.16R:** The results at LAR 2.16R (Figure 12) indicate little to no likelihood of erosion initiating and progressing into the levee at this location. This is a substantial reduction from the without project condition.



Figure 11. LAR 1.98R Berm design stochastic BSTEM results



Figure 12. LAR 2.16R Berm design stochastic BSTEM results

## Conclusions

Stochastic BSTEM models were developed for existing conditions for both existing topography and topography adjusted assuming computed bridge scour occurs. Scour was computed using conservative design equations from HEC-18 (FHWA 2012). The model results were provided to the risk cadre to assess the existing condition (baseline) risk for the project. The risk cadre utilized an erosion event tree, all available information including the BSTEM modeling, and expert opinion elicitation to determine the probability of levee breach prior to overtopping. The results of the risk assessment indicate that additional erosion countermeasures are needed for LAR 2.16R to meet project risk objectives. The design team considered the results of the risk assessment, BSTEM model results, and other information and concluded that no additional erosion countermeasures are needed at LAR 1.98R.

Stochastic BSTEM models were also developed for a proposed design of stone protection and a berm to reduce velocities at the site. The proposed stone protection design included a top of rock elevation that is lower than recommended by HEC 23 (FHWA 2009) guidelines. The summary of BSTEM results of implied likelihood of erosion initiating and progressing into the theoretical levee template is shown in Table 3 for the modeled scenarios.

		LAR 1.98R	LAR 2.16R
Without	No Scour	< or = 1%	< or = 25%
Project	With Scour*	100%	100%
With Stone Protection	No Scour	< 1%	10 - 25 %
With Berm	No Scour	1 -5 %	0%

Table 3. Summary of implied likelihood of erosion initiating and progressing into the theoretical levee prism from BSTEM Results

\*Likely conservatively high as it does not include a scour node in the event tree which would have a low probability because it is computed from a design equation.

The risk cadre used the BSTEM results and all available relevant information to determine the probability of levee breach prior to overtopping for the proposed design using expert opinion elicitation for existing conditions and with stone protection. The result of the draft risk assessment is that the stone protection does meet project risk objectives. But the risk cadre recommended the design team consider increasing the top of rock elevation upstream of LAR 1.98R to further reduce risk and increase confidence in the proposed design. The risk cadre has not yet evaluated the proposed berm design. However, BSTEM results for the berm design indicate it is likely to meet project risk reduction objectives as the likelihood of erosion initiation and progression to the levee is substantially reduced. Due to constructability challenges and expected positive performance, the design team has selected the berm design for construction. The berm design will be evaluated by the risk cadre in the future to ensure the design meets risk reduction objectives.

The design team was able to successfully incorporate scour from the complex flow field along the levee into the erosion estimates by accounting for potential bridge scour. Utilizing the results of BSTEM probabilistic erosion estimates to inform risk assessments allowed the design team to understand the existing and post-construction risk for the complex flow field at this site to optimize the design and reduce life safety risk.

#### References

- National Academies of Sciences, Engineering, and Medicine (NASEM) 2019. Relationship Between Erodibility and Properties of Soils. Washington, DC: The National Academies Press. <u>https://doi.org/10.17226/25470</u>.
- Rivas, T., Chowdhury, S., AuBuchon, J., Nguyen, H., Langendoen, E., Ursic, M., Eom, M., Majd,, M.S., Cheung, F.Y. 2019. "Erosion Assessment of Sacramento and American River Levees" Proc. of the 4th Joint Federal Interagency Conference on Sedimentation and Hydrologic Modeling, Reno, NV, pp. 118-132.
- Rivas, T., AuBuchon, J., Shidlovskaya, A., Langendoen, E., Work, P., Livsey, D., Timchenko, A., Jemes, K., Briaud, J.L. 2021a. "Risk-informed levee erosion countermeasure site selection and design in the Sacramento Area Part 1: Soil Sampling, testing, and data processing," Proc. 10<sup>th</sup> International Conference on Scour and Erosion, Virtual, pp 825 836.
- Rivas, T., AuBuchon, J., Shidlovskaya, A., Langendoen, E., Work, P., Livsey, D., Timchenko, A., Jemes, K., Briaud, J.L. 2021b. "Risk-informed levee erosion countermeasure site selection and design in the Sacramento Area Part 2: Probabilistic Numerical Simulation of Bank Erosion," Proc. 10<sup>th</sup> International Conference on Scour and Erosion, Virtual, pp 537 546.
- Simon, A., Pollen-Bankhead, N., and Thomas, R. E. (2011). "Development and application of a deterministic bank stability and toe erosion model for stream restoration," Stream Restoration in Dynamic Fluvial Systems: Scientific Approaches, Analyses, and Tools, A. Simon, S. J. Bennett, and J. M. Castro, eds, American Geophysical Union, Washington, DC, 453-474.
- Ursic M. and Langendoen E. 2021. Comparison of Test Methods for Erodibility of Bank Materials on the Lower American and Sacramento Rivers, adjacent to the City of Sacramento, California, Research Report No. 81. USDA, ARS, National Sedimentation Laboratory, Oxford, MS. "."
- USACE 1994. Engineer Manuel (EM) 1110-2-1601. Hydraulic Design of Flood Control Channels. U.S. Army Corps of Engineers, Washington, D.C.
- USACE 2019. Engineering and Construction Bulletin 2019-15, Interim Approach for Risk-Informed Designs for Dam and Levee Projects. U.S. Army Corps of Engineers, Washington, D.C.
- USDA. 2019. Bank Stability and Toe Erosion Model. U.S Department of Agriculture, Agriculture Research Service, Oxford, MS.
- FHWA. 2012. Hydraulic Engineering Circular No. 18 Evaluating Scour at Bridges. Federal Highway Administration, U.S. Department of Transportation, Washington, D.C.
- FHWA. 2009. Hydraulic Engineering Circular No. 23 Bridge Scour and Stream Instability Countermeasures: Experience, Selection, and Design Guidance-Third Edition. Federal Highway Administration, U.S. Department of Transportation, Washington, D.C.