Comparison of Measured Bedload with Predictions from Transport Equations in an Unarmored Ephemeral Channel

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Abstract

The Arroyo de los Pinos is a tributary of the Rio Grande that transports relatively coarse sediment into the river annually through flash flood events. This coarse-grained sediment can lead to problems for downstream infrastructure, such as sedimentation in reservoirs and increased channel maintenance requirements for flow conveyance. Over the past five years, a comprehensive database of bedload, suspended sediment, and meteorological-hydrologic measurements have been developed at the confluence of the channel to the Rio Grande. Bedload flux is monitored by three Reid-type slot samplers at 1-minute resolution, flow stage is continuously monitored with pressure transducers, and surface flow velocity is measured periodically using large scale particle imagery velocimetry (LSPIV) to produce a stage-discharge rating curve. Bed material samples have been collected and sieved, and channel geometry has been mapped in detail using drone imagery and structure from motion (SfM) photogrammetry. This dataset enables assessment of predicted bedload using a wide range of well-established equations, including the modified Einstein procedure, and the equations of Meyer-Peter and Müller, Wilcock and Crowe, and Parker. Crucially, we can compare the quality of prediction from these methods against the observed bedload transport at a range of flow depths between 5 – 50 cm (discharge at 0.25 – 10 m^3/s). The Pinos dataset provides an excellent opportunity to compare a range of transport equations and consider their relative performance in ephemeral, semi-arid, flash flood driven fluvial systems. Successful equation selection will enable the extension of our temporally-limited direct bedload measurements to approximate annual bedload yields from the Arroyos de los Pinos, as well as from other similar ephemeral tributaries to the Rio Grande and elsewhere. The best fitting bedload transport equations for the Arroyo de los Pinos are the Meyer-Peter and Müller and the Wilcock and Crowe equations.

Introduction

Motivation for Research

In the American southwest, approximately 80% of the rivers are ephemeral. Yet very little research has been conducted on ephemeral channels due to the difficulty of monitoring them, given the uncertainty of flow occurrence, as well as the danger that flash floods can pose. The Arroyo de los Pinos is an ephemeral tributary of the Rio Grande that transports thousands of tons of relatively coarse sediment into the river annually through flash flood events (Stark et al., 2021). If an excessive amount of sediment is deposited downstream it can lead to problems for infrastructure such as sedimentation in reservoirs and increased channel maintenance needed to ensure flow conveyance and prevent sediment plugs. The focus of this research is to evaluate existing bedload equations and determine how much bedload discharge is moved through our monitoring station. To estimate how much sediment is transported, several bedload equations have been analyzed through a web-based program called BedloadWeb that utilizes a specially built library of sediment transport functions for the R coding language (Recking, 2020). By comparing sediment transport equations to bedload collected with Reid-type slot samplers during flash floods, we narrow the numerous bedload equations available to those that best represent unarmored, ephemeral channels, i.e., those that best agree with data derived from several flow events. These equations could then be applied to other unarmored, ephemeral channels to estimate bedload flux, thereby assisting in resource allocation decisions for channel maintenance.

Research Goals

The broad objective of this research is to test the selected bedload equations against bedload flux data collected at the Pinos monitoring station. The goal is to narrow down the equations to those that best represent ephemeral channels and are representative across a variety of flow events. This analysis allows a comparison between classic bedload equations. Based on preliminary results and on previous research (Reid et al., 1995), the most promising bedload equations appear to be Wilcock and Crowe (2003) and Meyer-Peter and Müller (1948). These, along with two other well-known equations, the modified Einstein procedure (1950) and the Parker equation (1990), are explored in depth in this study.

Evaluating numerous bedload equations is also useful in that a sensitivity analysis can be conducted to determine which parameters in unarmored, ephemeral channels are the most influential. Based on preliminary research, one of the most significant parameters appears to be the grain size distribution of the channel bed (Laronne at el., 1994; Powell et al., 2003). This along with cross section size and stage are explored in this paper.

Background

Shear Stress and Shields Parameter

Sediment grain movement is affected by flow strength, the river's ability to supply grains for transport, and turbulence, among many other characteristics. This is observed in the movement of sand and gravel grains, due to the force of shear stress that is applied to individual grains until they are mobilized. As the grains are moved, they interact with other grains and cause instability in the channel that results in increased sediment movement, at least until the river competence decreases and the grains settle on the bed again (Charbonneau, 2017).

Boundary shear stress is a reliable metric that represents mobilization of a given grain. In the case of rivers, forces acting on the channel banks and channel bed must be considered. This is typically represented by a combination of drag forces caused by the bed grain roughness, bed forms, bars, and bends in the channel (Wilcock & Kenworthy, 2002). Average bed shear stress for a cross section is often approximated as:

$$\tau = g\rho_w R_h S \tag{1}$$

where τ is average cross sectional bed shear stress, g is gravity, ρ_w is the density of water, R_h is hydraulic radius, and S is the energy slope. Bedload flux is estimated by many equations as a function of the excess shear above some critical value τ_c. A similar approach using a non-dimensionalized version of τ was developed by Shields (1936):

$$\tau^* = \frac{\tau}{g(\rho_s - \rho_w)D_{84}} \tag{2}$$

where τ^* is the non-dimensional Shields parameter, ρ_s is the density of the sediment, and D_{84} is the diameter of the 84th percentile bed material grain size. Using the dimensional shear stress, many equations estimate bedload flux as a function of the excess Shields parameter above a critical value τ^*_c. The Meyer-Peter and Müller (1948) equation is a classic example of this form (Wong and Parker, 2006).

Another approach that has shown promise in bedload estimation is to treat each grain size fraction individually and sum the results. Einstein (1950) was one of the first to attempt this. Parker (1990) developed an equation that focused on those size fractions at the bed surface. Wilcock & Crowe (2003) simplified this concept to a two-fraction (sand and gravel) approach that retained the main benefits but reduced the computational expense.

Summary of Equations Tested

This paper focuses on four bedload transport equations: Meyer-Peter and Müller (MPM), Parker 90, Wilcock and Crowe (W&C), and Einstein-Brown.

The Meyer-Peter and Müller equation:

$$\Phi = 8\left[\left(\frac{n'}{n}\right)^{3/2} \tau_* - 0.047\right]^{\frac{3}{2}} \tag{3}$$

where Φ is a dimensionless transport rate, *n'/n* is used to correct shear stress, and τ_* is the Shields parameter for the median diameter. This equation was tested in the field, and with laboratory experiments with both uniform and non-uniform materials (Recking, 2020).

The Parker 90 equation:

$$W_i^* = \frac{(s-1)gq_{vi}}{F_i u_{*s}^3} = 0.00218\,G(\Phi) \tag{4}$$

with

$$\Phi = \omega\Phi_{sgo}\left(\frac{D_i}{D_g}\right)^{-0.0951}, \Phi_{sgo} = \frac{\tau_g^*}{\tau_{ssrg}^*}, \tau_g^* = \frac{\tau}{\rho(s-1)gD_g}, \tau_{ssrg}^* = 0.0386 \tag{5}$$

$$G(\Phi) = \begin{cases} 5474\left(1 - \frac{0.853}{\Phi}\right)^{4.5}, \text{ for } \Phi > 1.59 \\ exp[14.2(\Phi - 1) - 9.28(\Phi - 1)^2], \text{for } 1 \leq \Phi \leq 1.59 \\ \Phi^{14.2}, \text{for } \Phi < 1 \end{cases} \tag{6}$$

$$\omega = 1 + \frac{\sigma}{\sigma_o(\Phi_{sgo})}\left[\omega_o\left(\Phi_{sgo}\right) - 1\right] \tag{7}$$

where *s* is relative density, *g* is gravity, q_{vi} is volumetric transport per unit width, for grain size *i*, u_* is shear velocity, D_x is grain diameter, *τ* is shear stress, τ_* is Shields parameter, *ρ* is fluid density, *Φ* is dimensionless transport rate. Values of $\sigma_o\left(\Phi_{sgo}\right)$ and $\omega_o\left(\Phi_{sgo}\right)$ are found by visual inspection of a figure from the Parker (1990) paper. This equation was specifically developed for bedload of gravel size and coarser; sand should be removed from the grain size curve. The original data used to parameterize this equation derives from Oak Creek (Recking, 2020).

The Wilcock and Crowe equation:

$$W_i^* = \begin{cases} 0.002\Phi^{7.5}, \ \Phi < 1.35 \\ 14\left(1 - \frac{0.894}{\Phi^{0.5}}\right)^{4.5}, \ \Phi \geq 1.35 \end{cases} \quad (8)$$

with

$$W_i^* = \frac{(s-1)gq_{vi}}{f_i u_*^3}, \ \Phi = \frac{\tau}{\tau_{ri}}, \tau_{ri} = \tau_{rg}\left(\frac{D_i}{D_g}\right)^b, b = \frac{0.67}{1+\exp\left(1.5-\frac{D_i}{D_g}\right)} \quad (9)$$

$$\tau_{rg} = (s-1)\rho g D_g[0.021 + 0.015\exp(-20F_s)] \quad (10)$$

The parameters include Φ as dimensionless transport rate, s_i is relative density of class i, g is gravity, q_{vi} is volumetric transport per unit width, f_i is the Darcy-Weisbach friction coefficient for class i, u is shear velocity, D_i is grain diameter of class i, D_g is geometric mean diameter of the bed surface, and F_s is the fraction of sand. This equation was parameterized using data from flume experiments with slopes in the range 0.1 - 1.8%, accounting for both sand and gravel grain sizes (Recking, 2020).

The Einstein-Brown equation:

$$\Phi = \left[\sqrt{\frac{2}{3} + \frac{36v^2}{g(s-1)D^3}} - \sqrt{\frac{36v^2}{g(s-1)D^3}}\right] F(\tau_*) \quad (11)$$

$$F(\tau_*) = 2.15\exp\left(-\frac{0.391}{\tau*}\right) \text{ for } \tau_* < 0.18 \quad (12)$$

$$F(\tau_*) = 40\,\tau_*^3 \ for\ \tau_* > 0.18 \quad (13)$$

where D is the grain diameter. The Einstein-Brown equation was created in 1950 and is based on laboratory data from Gilbert (1914) and Meyer-Peter and Müller (1948) data (Recking, 2020).

Study Site Description and Methods

The Arroyo de los Pinos is near Socorro, NM, at the northern edge of the Chihuahuan Desert and has a drainage area of 32 km^2. The geology of the watershed is primarily made up of sandstone, limestone, mudstone, and alluvium. This tributary is dry most of the year, flowing for brief intervals during monsoon season, which occurs from July to September.

To monitor flow events, the site has a network of pressure transducers to measure stage, three Reid-type slot samplers to continuously monitor bedload flux and collect bedload sediment, automated pump samplers to collect suspended sediment samples, a video camera to measure surface flow velocity using large scale particle image velocimetry, and seismic nodes and broadbands to monitor ground motions caused by bedload transported.

The three Reid-type slot samplers installed at the Pinos site are essentially covered pits with 11-cm-wide slot openings. The concrete pits each hold a steel box that rests on a pressure pillow. As sediment moving over the opening falls in, the added weight on the pressure pillow is recorded at 1-minute intervals. The samplers can hold approximately 0.5 m^3 of sediment before filling. Due to the high rates of sediment transported during higher flow events, these samplers typically fill in the first 15-30 minutes of a flow event, unless it is a very low-stage flow. These high-temporal-resolution bedload data allow comparing model-based prediction with actual monitored bedload fluxes.

Stage is monitored at the site by three vented pressure transducers, one in each slot sampler. Two unvented pressure transducers are installed in wells 80 m and 230 m upstream of the slot samplers. Discharge from a calibrated rating curve was calculated at a 1-minute resolution for the controlled cross section of the bedload station. Stage values at the upstream wells were recorded every two to three minutes (depending on the year). The upper well cross section was used in all hydraulic parameter estimates, because installation of the bedload monitoring equipment altered the channel geometry. The station has produced reliable discharge estimates; these were applied to the unaltered upstream cross section using the Manning equation.

Grain size distributions were determined for bed material in the thalweg and bars, at both the surface and the subsurface. Surface samples were taken by shoveling the upper 5 cm of a bar or thalweg. The subsurface was sampled 5 cm below the surface of a bar or thalweg. The sampled sediment was allowed to dry at least two weeks. The samples were thereafter sieved in the range of 90 mm to 0.063 mm and weighed.

Cross sections for these models were created by using a RTX-GPS surveying system. This equipment uses real-time corrected GPS satellite observations to determine point locations at a precision of ~ 3-5 cm, which is accurate enough for the sake of this research.

Most of the analysis presented here was done in BedloadWeb, a web-hosted R-based calculation toolbox that takes site-specific input data and calculates estimates from numerous bedload equations (Recking, 2020). The required inputs are: grain size distribution of the channel, a cross section of the channel where hydraulic data is captured, discharge time series, and slope. Roughness was calculated from the grain size data using Ferguson’s (2003) equation. If stage data and discharge data are available, the accuracy of the calculated stage-discharge relationship and the bedload estimates can be evaluated.

Results

The grain size distribution is a key determinant of bedload discharge. Bedload estimates were tested using four different grain size distributions: thalweg surface, bar surface, average of thalweg and bar surface, and average of thalweg surface and subsurface. Focusing on the surface of the thalweg and bar, six samples were analyzed from each of these locations to determine the variability in grain size distribution. The bars are significantly coarser than the thalweg (Figure 1).

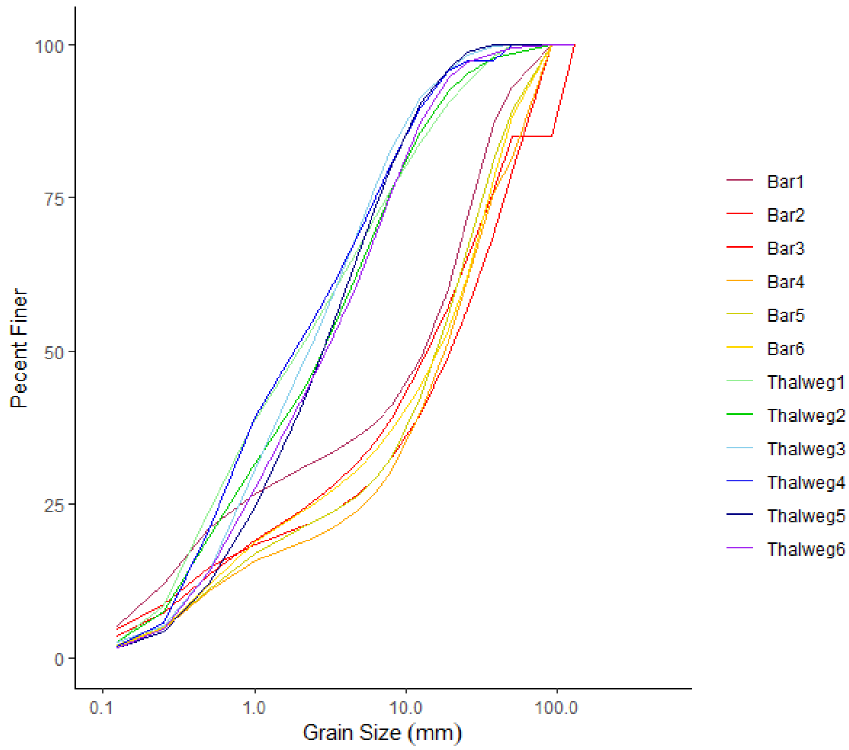

Figure 1. Grain size distribution of bar and thalweg surface samples in the Arroyo de los Pinos

The hydraulic component of BedloadWeb was evaluated by comparing the stage values measured during the flow events to the estimated stage values calculated from the discharge hydrographs, Manning-based velocity estimates, and channel cross section geometry. The stage data from the three flow events we tested, especially the July 5th event, line up well with the estimate from BedloadWeb (Figure 2).

With the hydraulic data, grain size data, and cross-sectional geometry, bedloadweb calculates bedload discharge using the four methods being tested (as well as nearly a dozen others). Estimated bedload fluxes are compared against the measured bedload data, showing that the calculated values using the thalweg surface grain size distribution agreed much better with the field data than those calculated using the bar surface grain size data (Figures 3 through 5). This is consistent with the observation that two of the three flow events were small ones with stages that barely reached the bars, if at all. The Einstein-Brown method begins to significantly overestimate the observations for the thalweg grain size distribution for the highest fluxes measured (Figure 3). The other three equations perform similarly at the high fluxes. At the lower fluxes, Wilcock and Crowe and Einstein-Brown agree more closely with the field data.

When all four equations and all four grain size distributions are compared, the bedload data best lines up with the Meyer-Peter and Müller and the Wilcock and Crowe equations using the thalweg surface grain distribution (Figure 6).

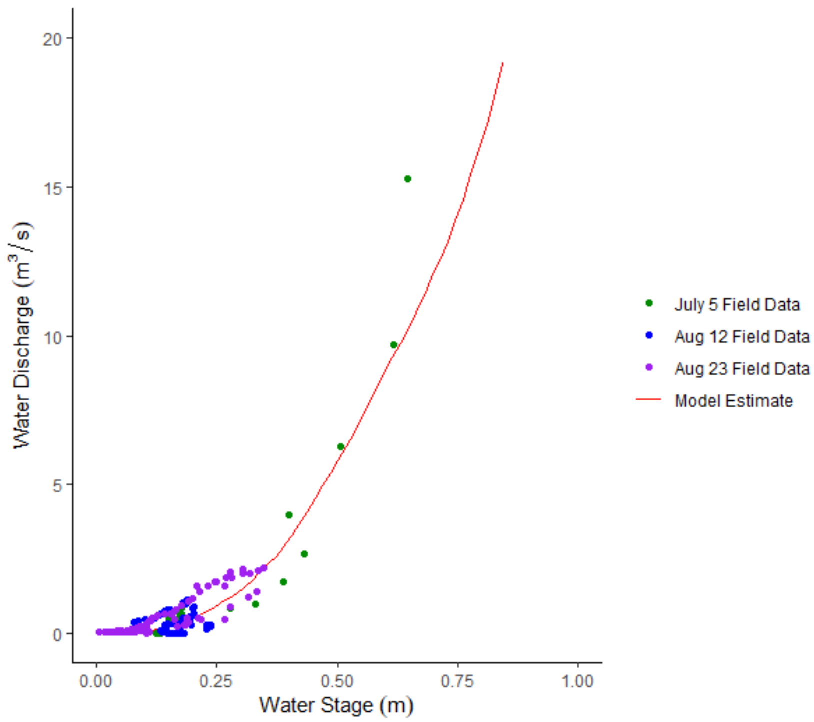

Figure 2. Stage-discharge curve calculated by BedloadWeb based on field data

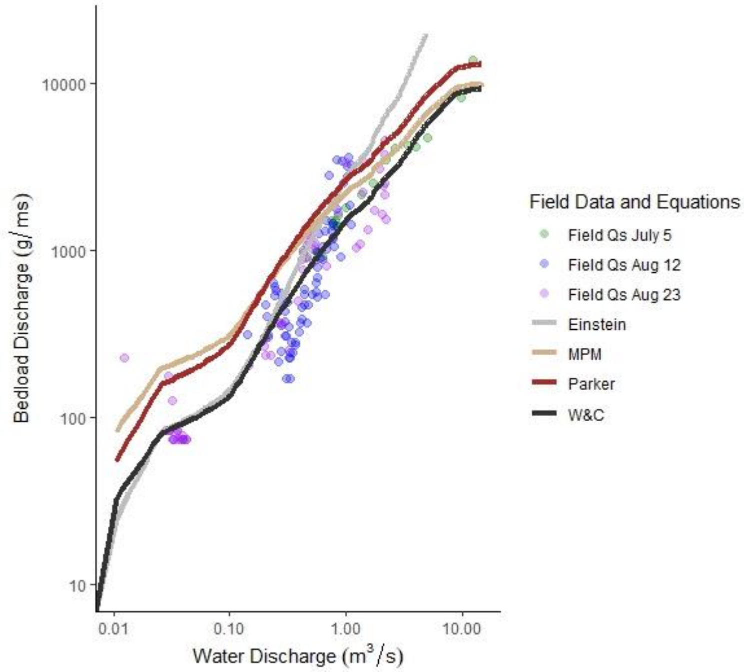

Figure 3. Thalweg surface-based bedload discharge as a function of water discharge for each flow event with equation estimates and a line of best fit for each of the three flow events

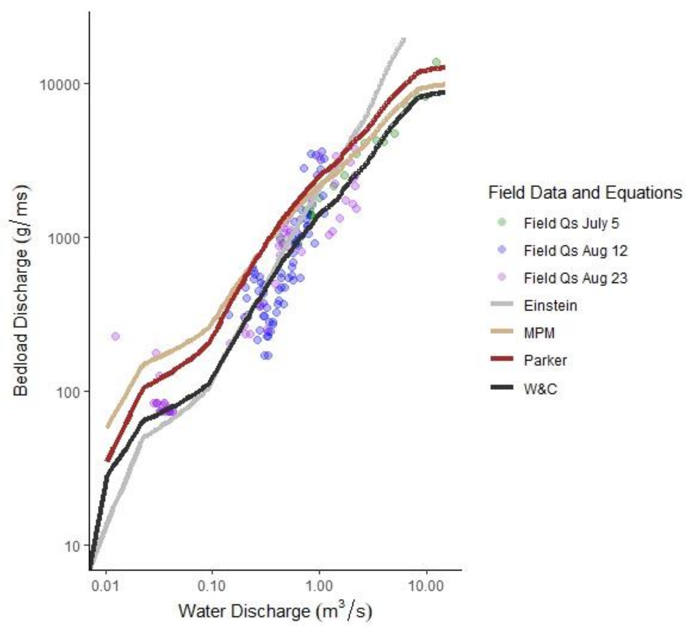

Figure 4. Thalweg surface and subsurface-based bedload discharge as a function of water discharge for each flow event with equation estimates and a line of best fit for each of the three flow events

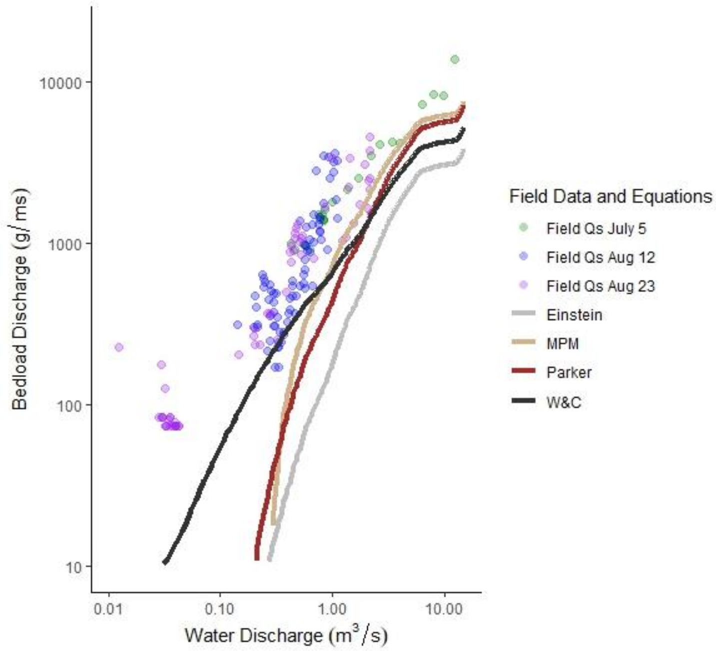

Figure 5. Bar surface-based bedload discharge as a function of discharge for each flow event with equation estimates and a line of best fit for each of the three flow events

The total bedload transported during each flow event could not be measured in the field because of filling of the slot samplers. However, each bedload equation can be used to calculate total bedload transport for each flow event by integrating the bedload flux estimates over the time series of the entire event hydrograph (Figures 7). The MPM, W&C, and Parker 90 equations agreed well with regard to the total transport, with the Einstein-Brown being a high outlier. Event-total bedload estimates are reported in metric tons below (Table 1).

Table 1. Bedload estimates for each flow event and equation in metric tons

	Einstein-Brown	MPM	Parker 90	W&C
July 5th 2021	9,490	1,460	1,270	1,440
August 12th 2021	2,210	2,590	370	1,530
August 23rd 2021	100	70	20	50

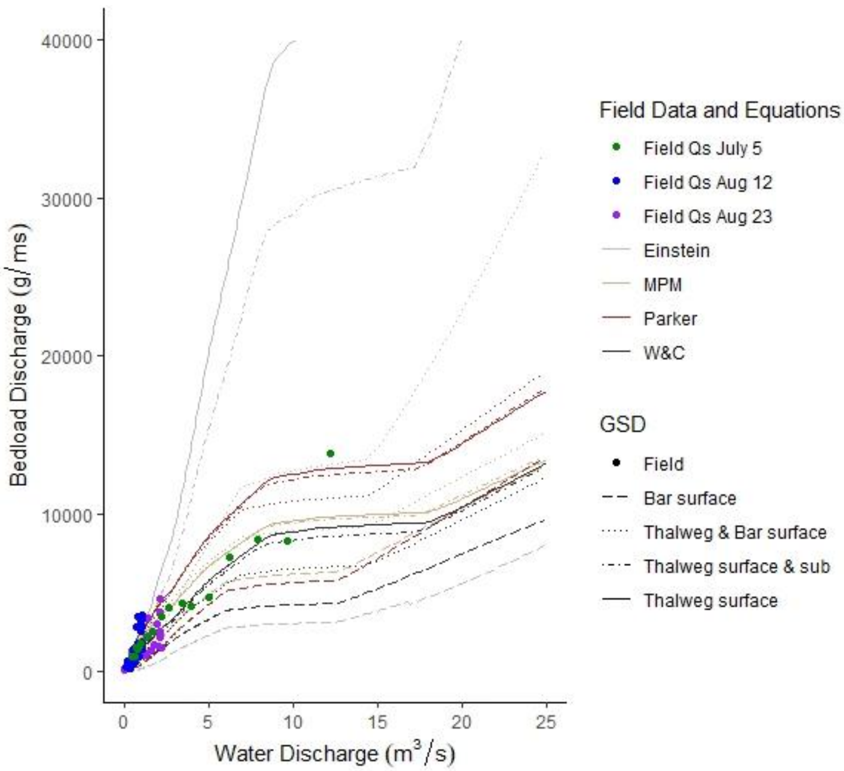

Figure 6. Bedload equations and field-derived bedload data plotted against water discharge. The equations are plotted for each of the channel bed grain size distributions

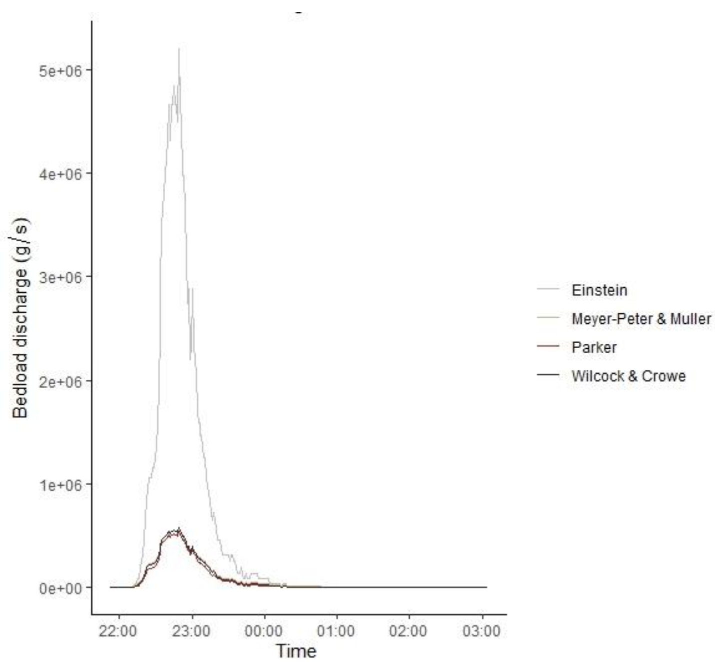

Figure 7. Temporal variation of bedload discharge according to the four equations

Discussion

The thalweg grain size distribution is much finer than that of the bars (Figure 1). This is due to the fact that there is no winnowing effect in ephemeral channels, and the bars are only inundated in the largest events, when sediment transport is high and even the largest channel material is in motion. However, as flow increases and the bars are covered, sediment can be transported over the bars as well. If a large sediment grain is being transported by the flow, it may also be more prone to settle if it travels over a bar due to the reduced flow depth and greater roughness, hence the reduced capacity to carry it. A relevant observation from our three sediment samplers is that bedload flux varied across the channel depending on the upstream bed topography. Bedload flux was higher at locations where the thalweg was directing transport from upstream, whereas if a bar formed upstream of a location, sediment was deflected by the shallower depth and coarser bed material (Stark et al., 2021). This is analogous to some previously documented cases, such as shallow overland flow events, wherein connectivity and movement of sediment is dominated by microtopography, and higher elevation areas remain unaffected during a flow event (Okin et al., 2015). Hence, sampling bedload needs to represent cross sectional variations.

Predictions of water discharge based on BedloadWeb match the observed data (Figure 2). Values from the August 12th and August 23rd flow events have a large range around the base of the curve, as small errors in pressure transducer placement may have led to skewed measurements that are more obvious at these shallower flows. Data from the larger flood of July 5th fits the model prediction well.

The thalweg surface shows that the bedload data from the flow events fit the Wilcock and Crowe and Meyer-Peter and Müller equations best, with the Einstein-Brown equation also being a good estimate to about 0.8 m^3/s (Figure 3). Similar results occur with the use of the average of the thalweg surface and subsurface grain size distributions in the equations (Figure 4), as the grain size distribution of the thalweg does not vary considerably with shallow depths. When assuming sediment distributions characteristic of the bar surface, the data does not fit the curves as well (Figure 5) as the pit traps were filled long before the bars were activated. The estimates line up somewhat with the July 5th event, as it was a relatively large and flashy flood. These results confirm our observations in the field – most of the bedload data from the Arroyo de los Pinos took place during relatively shallow events, when the majority of sediment derived from the channel thalweg, as found elsewhere (Laronne et al., 1994).

The bedload data best line up with the Meyer-Peter and Müller and the Wilcock and Crowe equations and favor the thalweg GSDs (Figure 6). However, our highest single field bedload flux datum fits best with the Parker 90 equation. The relative performance of the equations may continue shifting once we obtain flux measurements at higher shear stresses. Based on Figures 3 - 6, the best bedload transport equation across the entire range of data for ephemeral unarmored channels is the Wilcock and Crowe equation, followed by the Meyer-Peter and Müller equation.

Einstein-Brown is an outlier in the calculation of bedload discharge (Figure 7). It is unsurprising that it differs from the other equations because it was intended to be used primarily in sand-bed channels with important interactions between bedload and suspended load. The mixed sand-and-gravel character of our channel appears to be poorly suited to its application, especially as flow and sediment flux increase beyond mere sand transport. Note that the Einstein-Brown equation does relatively well when flows are shallow and bedload is limited to the sand component of the channel bed.

Conclusions

Based on the results above, the most appropriate equations for bedload transport in the unarmored, mixed sand-and-gravel ephemeral channels are the Wilcock and Crowe and Meyer-Peter and Müller equations. These equations worked well when given the thalweg grain size distribution data, whereas the bar grain size data produced a weaker correlation with field data. This is due to the bedload being sampled at lower stages, that did not yet inundate the bars, and therefore predicted a coarser grain size than the actual sediment in transport which resulted in under-predictions of transport rates. This demonstrates that the grain size distribution is a primary control on the accuracy of modeled sediment transport. Stage is another important control, but it is measured to an adequate level of accuracy. Our preliminary results may be used in other ephemeral channels. However, we recommend using unit discharge (q = Q/w) for BedloadWeb, thereby allowing comparison of bedload flux (in mass flux per width) to unit discharge of water, which better generalizes to other relevant rivers.

References

Charbonneau, P. (2017). Natural complexity - A modeling handbook. Princeton University Press.

Einstein, H. A. (1950). The bed-load function for sediment transportation in open channel flows. https://naldc.nal.usda.gov/download/CAT86201017/PDF

Ferguson, R. I. (2003). Emergence of abrupt gravel to sand transitions along rivers through sorting processes. *Geology*, 31(2), 159. https://doi.org/10.1130/0091-7613(2003)031<0159:eoagts>2.0.co;2

Laronne, J.B., Reid. I., Yitshak, Y. and Frostick, L.E., 1994. The non-layering of gravel streambeds under ephemeral flood regimes. *Journal of Hydrology*, 159, 353-363, https://doi.org/10.1016/0022-1694(94)90266-6.

Meyer-Peter, E., & Müller, R. (1948). "Formulas for Bed-Load Transport," Proc. 2nd Meeting of the IAHR, Stockholm, Sweden, pp 39–64.

Okin, G. S., Heras, M. M.-de, Saco, P. M., Throop, H. L., Vivoni, E. R., Parsons, A. J., Wainwright, J., & Peters, D. P. C. (2015). Connectivity in dryland landscapes: Shifting concepts of spatial interactions. *Frontiers in Ecology and the Environment*, 13(1), 20–27. https://doi.org/10.1890/140163

Parker, G. (1990). Surface-based bedload transport relation for Gravel Rivers. *Journal of Hydraulic Research*, 28(4), 417–436. https://doi.org/10.1080/00221689009499058

Powell, D.M., Laronne, J.B. and Reid, I., 2003. The dynamics of bedload sediment transport in ephemeral gravel-bed rivers. *Advances in Environmental Monitoring and Modelling*, http://www.kcl.ac.uk/advances

Recking, A. (2020, January 1). BedloadWeb Concepts and Equations for bedload computation. https://en.bedloadweb.com/ . Retrieved November 19, 2022, from https://en.bedloadweb.com/The%20equations.pdf

Reid, I., & Laronne, J. B. (1995). Bed load sediment transport in an ephemeral stream and a comparison with seasonal and perennial counterparts. *Water Resources Research*, 31(3), 773–781. https://doi.org/10.1029/94wr02233

Shields, A. F. (1936). Application of similarity principles and turbulence research to bed-load movement. Turbulence bed-load movement - authors.library.caltech.edu. Retrieved December 7, 2022, from https://authors.library.caltech.edu/25992/1/Sheilds.pdf

Stark, K., Cadol, D., Varyu, D., & Laronne, J. B. (2021). Direct, continuous measurements of ultra-high sediment fluxes in a sandy gravel-bed ephemeral river. *Geomorphology*, 382, 107682. https://doi.org/10.1016/j.geomorph.2021.107682

Wilcock, P. R., & Kenworthy, S. T. (2002). A two-fraction model for the transport of sand/gravel mixtures. *Water Resources Research*, 38(10). https://doi.org/10.1029/2001wr000684

Wilcock, P. R., & Crowe, J. C. (2003). Surface-based transport model for mixed-size sediment. *Journal of Hydraulic Engineering*, *129*(2), 120–128. https://doi.org/10.1061/(asce)0733-9429(2003)129:2(120)

Wong, M., &; Parker, G. (2006). Reanalysis and correction of bed-load relation of Meyer-Peter and Müller using their own database. *Journal of Hydraulic Engineering*, 132(11), 1159–1168. https://doi.org/10.1061/(asce)0733-9429(2006)132:11(1159)